

# Deep Ocean Mixing

Raffaele Ferrari

Earth, Atmospheric and Planetary Sciences, MIT

Les Houches, August 2017

A place on earth more awesome  
than anything in space.

THE  
ABYSS

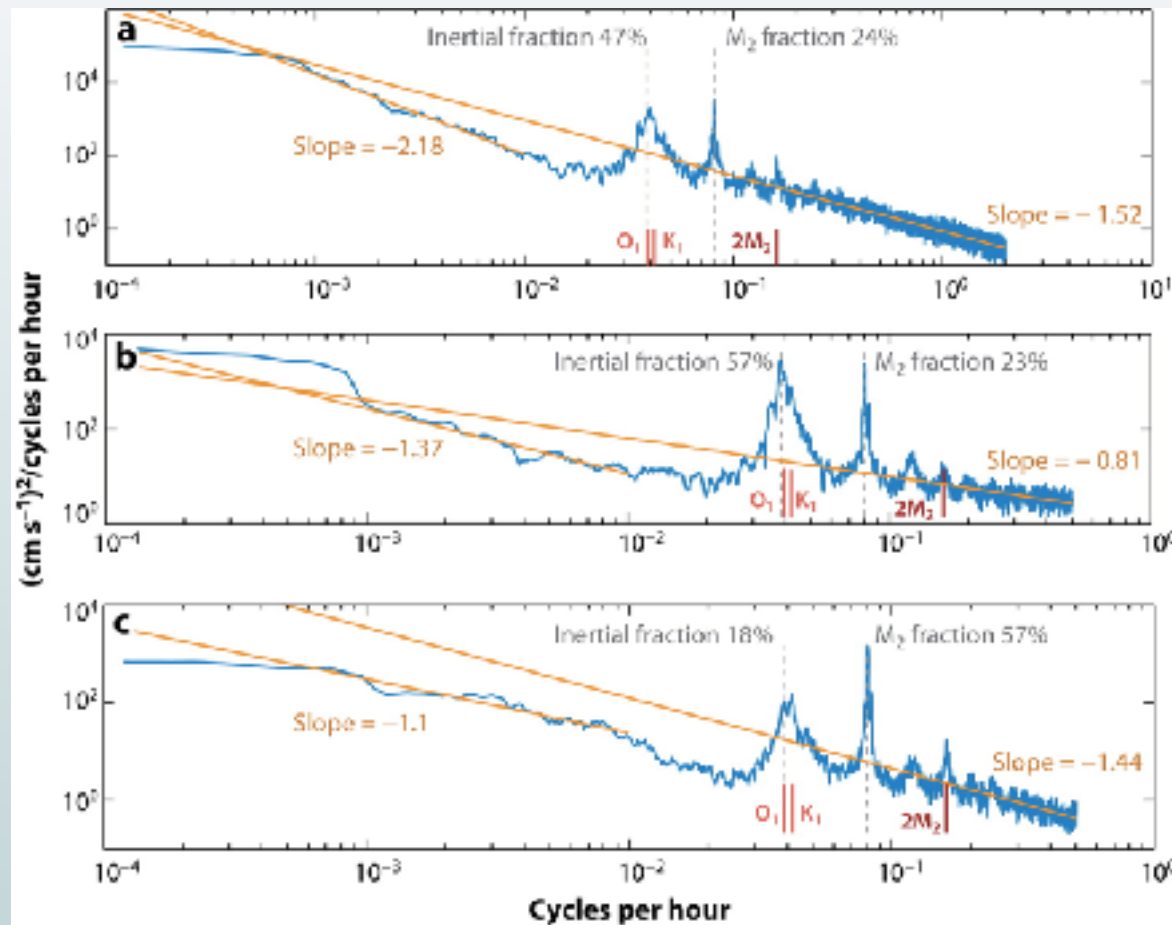
The image is a movie poster for 'The Abyss'. It features a dark, blue-toned underwater scene. At the bottom center, a bright, glowing light source, possibly a sun or a powerful lamp, illuminates the surrounding water and creates a lens flare effect. The light rays spread outwards, highlighting the rough, rocky textures of the cave walls on either side. The overall atmosphere is mysterious and awe-inspiring.


# Outline

- ▶ Turbulence and diapycnal mixing
- ▶ Methods used to estimate ocean diapycnal mixing
  - temperature variance budget
  - turbulent kinetic energy budget
  - tracer release experiments
- ▶ Direct estimates of ocean diapycnal mixing
  - North Atlantic (shallow)
  - Brazil Basin (deep)
  - Drake Passage (mid-depth)

# Turbulence and mixing

# Turbulence in ocean interior



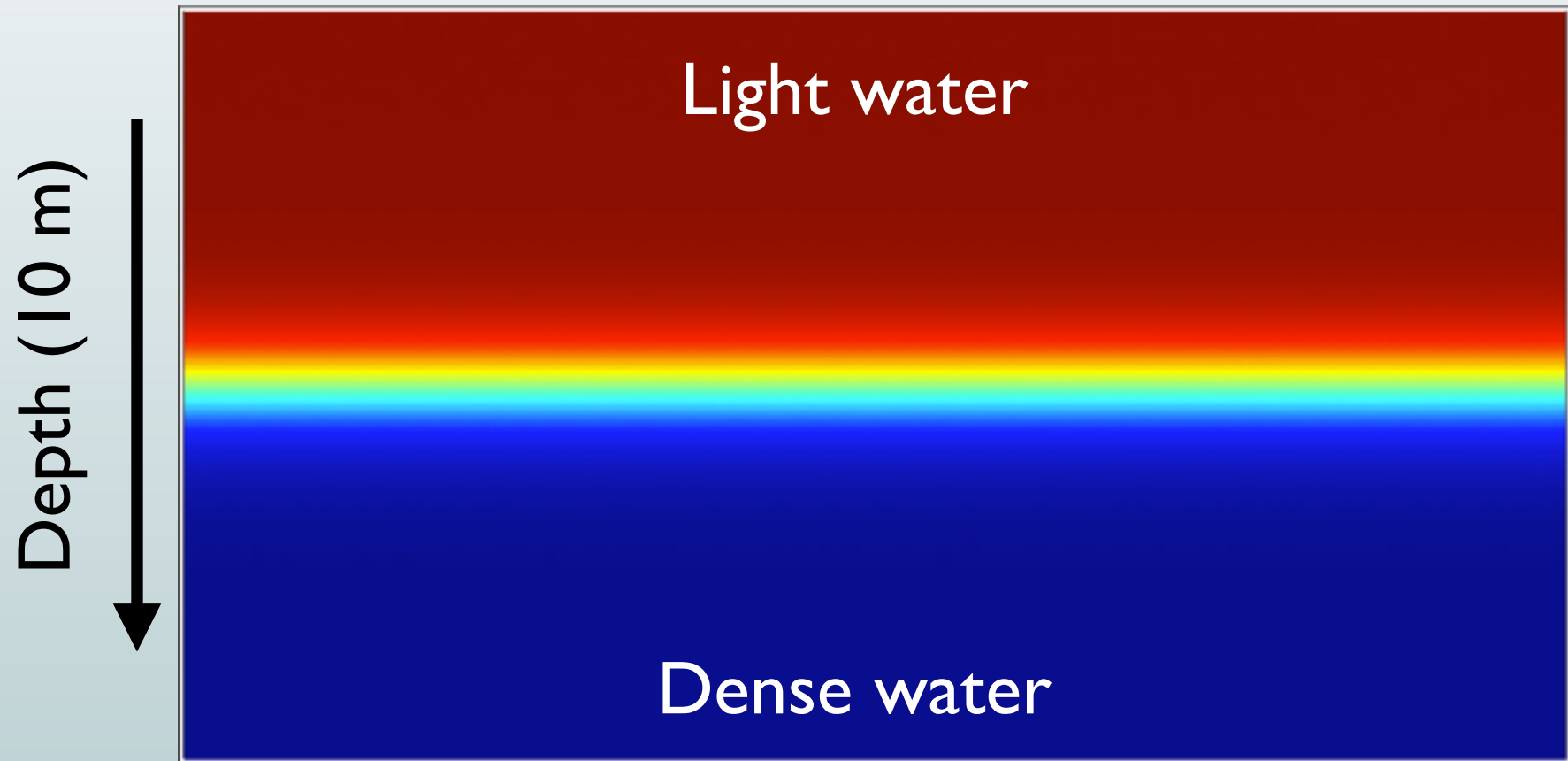
 Ferrari R, Wunsch C. 2009.  
Annu. Rev. Fluid Mech. 41:253–82

Subinertial motions have large  $Ri$  and small  $Ro$

Superinertial motions can develop small  $Ri$  and large  $Ro$



# Breaking internal waves



# Turbulent buoyancy fluxes

# Boussinesq Equations



$$\partial_t \mathbf{u} + (\mathbf{u} \cdot \nabla) \mathbf{u} + f \hat{\mathbf{z}} \times \mathbf{u} = -\rho_0^{-1} \nabla p + b \mathbf{z} + \nabla \cdot (\nu \nabla \mathbf{u}),$$

$$\nabla \cdot \mathbf{u} = 0,$$

$$\partial_t \theta + (\mathbf{u} \cdot \nabla) \theta = \nabla \cdot (\kappa_\theta \nabla \theta),$$

$$\partial_t S + (\mathbf{u} \cdot \nabla) S = \nabla \cdot (\kappa_S \nabla S),$$

$$b = g\alpha(\theta - \theta_0) - g\beta(S - S_0), \quad \rho = \rho_0 (1 - g^{-1}b)$$

$$\implies \partial_t b + (\mathbf{u} \cdot \nabla) b = \nabla \cdot (\kappa_\theta g\alpha \nabla \theta - \kappa_S g\beta \nabla S)$$

# Reynolds decomposition



Buoyancy budget

$$\partial_t b + (\mathbf{u} \cdot \nabla) b = \nabla \cdot (\kappa_\theta g \alpha \nabla \theta - \kappa_S g \beta \nabla S)$$

Subinertial buoyancy budget

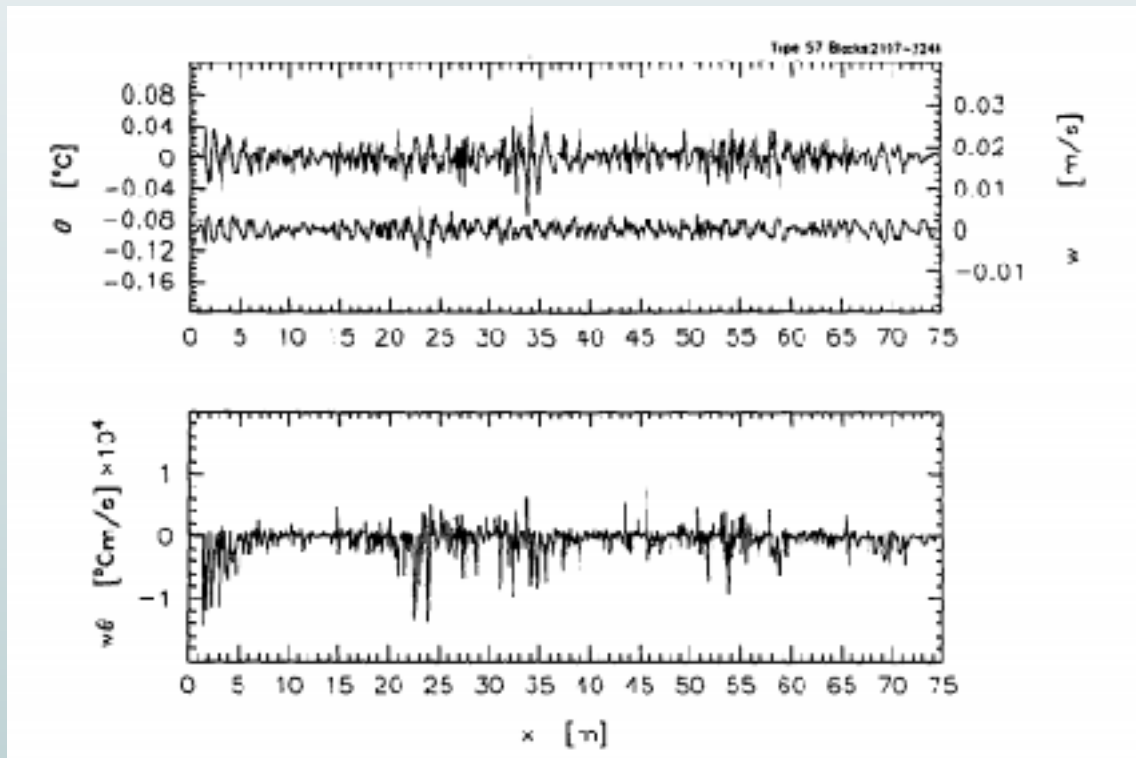
$$\begin{aligned} \partial_t \bar{b} + (\bar{\mathbf{u}} \cdot \nabla) \bar{b} &= -\nabla \cdot (\overline{\mathbf{u}'b'} - \kappa_\theta g \alpha \nabla \bar{\theta} + \kappa_S g \beta \nabla \bar{S}) \\ &\simeq -\nabla \cdot \overline{\mathbf{u}'b'} \\ &\simeq -\partial_z \overline{w'b'} \end{aligned}$$

# Measurements of turbulent buoyancy fluxes



# I. Direct eddy correlations

- Compute  $w$  and  $b$  ( $\theta$  and  $S$ )
- High pass signals to extract super-inertial signals  $w'$  and  $b'$
- Compute correlations by averaging over time  $\overline{w'b'}$



*Fleury and Lueck, 1994*

- Challenge I: measure  $w$  and  $b$  ( $\theta$  and  $S$ ) at same location from stable platform
- Challenge II : average over long enough time

# II. Temperature variance budget

- Potential temperature budget

$$\partial_t \theta + (\mathbf{u} \cdot \nabla) \theta = \nabla \cdot (\kappa_\theta \nabla \theta)$$

- Mean and eddy potential temperature budget

$$\partial_t \bar{\theta} + (\bar{\mathbf{u}} \cdot \nabla) \bar{\theta} = \nabla \cdot (-\overline{\mathbf{u}'\theta'} + \kappa_\theta \nabla \bar{\theta})$$

$$\partial_t \theta' + ((\bar{\mathbf{u}} + \mathbf{u}') \cdot \nabla) \theta' + \mathbf{u}' \cdot \nabla \bar{\theta} = \nabla \cdot (\overline{\mathbf{u}'\theta'} + \kappa_\theta \nabla \theta')$$

- Potential temperature variance budget

$$\frac{1}{2} \partial_t \overline{\theta'^2} + \frac{1}{2} \nabla \cdot \left[ \overline{(\bar{\mathbf{u}} + \mathbf{u}')\theta'^2} - \kappa_\theta \nabla \overline{\theta'^2} \right] = -\overline{\mathbf{u}'\theta'} \cdot \nabla \bar{\theta} - \kappa_\theta \overline{|\nabla \theta'|^2}$$

# II. Temperature variance budget

- Assuming that turbulence is stationary, homogeneous and isotropic:

$$\begin{aligned}
 \cancel{\frac{1}{2} \partial_z \overline{\theta'^2}} + \cancel{\frac{1}{2} \nabla \cdot [(\bar{\mathbf{u}} + \mathbf{u}') \theta'^2]} - \cancel{\kappa_\theta \nabla \overline{\theta'^2}} &= -\overline{\mathbf{u}' \theta'} \cdot \nabla \bar{\theta} - \kappa_\theta \overline{|\nabla \theta'|^2} \\
 &\quad - 3 \kappa_\theta \overline{(\partial_z \theta')^2}
 \end{aligned}$$

stationarity
homogeneity
isotropy

- Assuming that  $\nabla \bar{\theta} \simeq \partial_z \bar{\theta} \hat{\mathbf{z}}$

$$\overline{w' \theta'} \simeq - \frac{3 \kappa_\theta \overline{(\partial_z \theta')^2}}{\partial_z \bar{\theta}} \equiv - \frac{1}{2} \frac{\chi}{\partial_z \bar{\theta}}$$

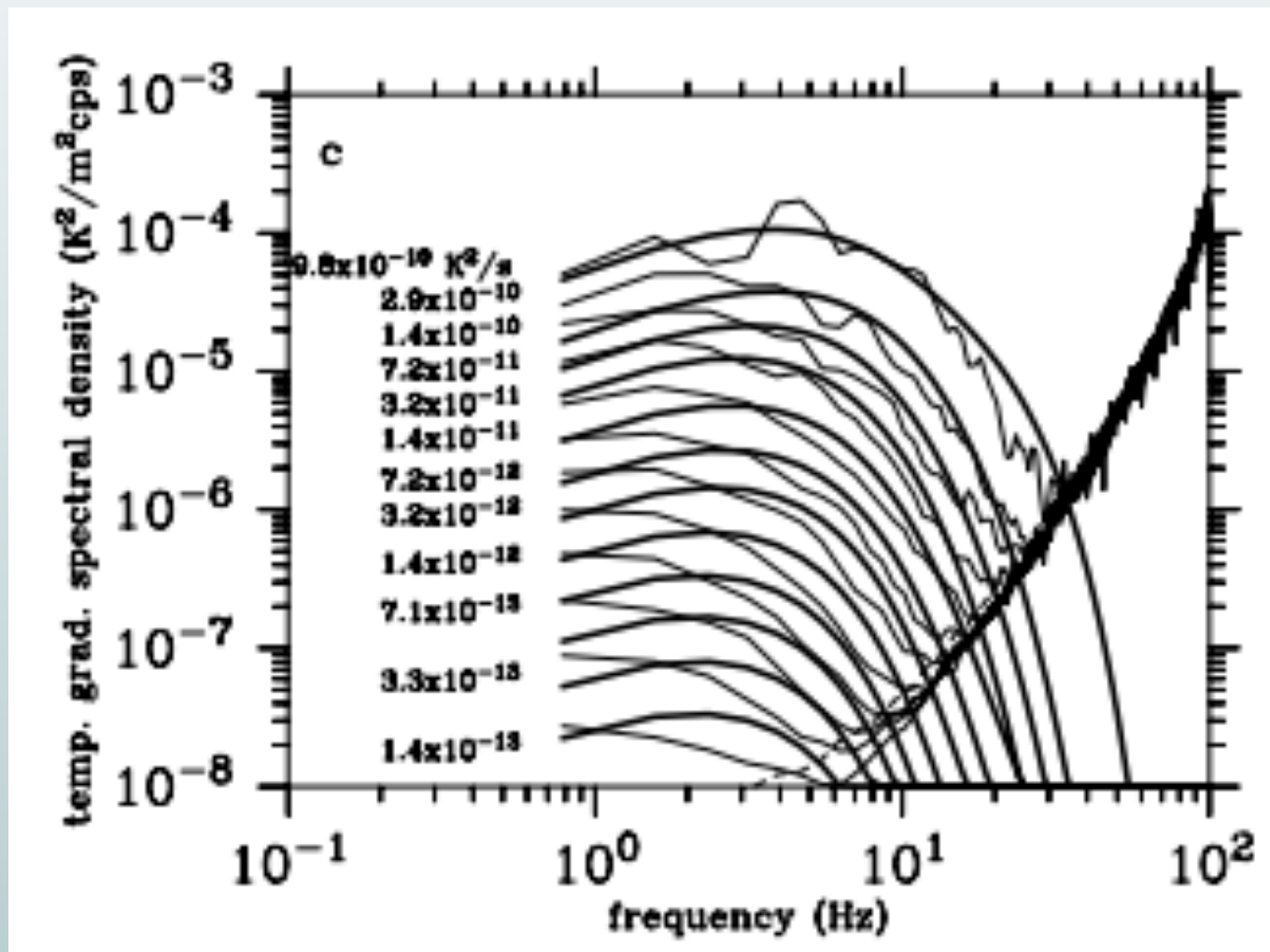
- Assuming that temperature dominates buoyancy gradients

$$\overline{w' b'} \simeq g \alpha \overline{w' \theta'} \simeq - \frac{g \alpha}{2} \frac{\chi}{\partial_z \bar{\theta}} \quad (\text{Osborn-Cox, 1972})$$



# Microstructure thermistors

Temperature gradient spectra ( $\chi$ )





# III. Kinetic energy budget

- Full turbulent kinetic energy budget

$$\begin{aligned} \frac{1}{2} \partial_t \overline{|\mathbf{u}'|^2} + \frac{1}{2} \nabla \cdot \left[ \overline{(\bar{\mathbf{u}} + \mathbf{u}') |\mathbf{u}'|^2} - \nu \nabla \overline{|\mathbf{u}'|^2} + \rho_0^{-1} \overline{p' w'} \right] = \\ = -\overline{\mathbf{u}' \mathbf{u}'} \cdot \nabla \bar{\mathbf{u}} + \overline{w' b'} - \nu \overline{|\nabla \mathbf{u}'|^2} \end{aligned}$$

- Turbulent kinetic energy budget for stationary, homogeneous, isotropic turbulence

$$-\overline{\mathbf{u}' \mathbf{u}'} \cdot \nabla \bar{\mathbf{u}} + \overline{w' b'} \simeq \frac{15}{2} \nu \overline{(\partial_z u')^2}$$

- Assuming that  $\nabla \bar{\mathbf{u}} \simeq \partial_z \bar{\mathbf{u}}_h \hat{\mathbf{z}}$

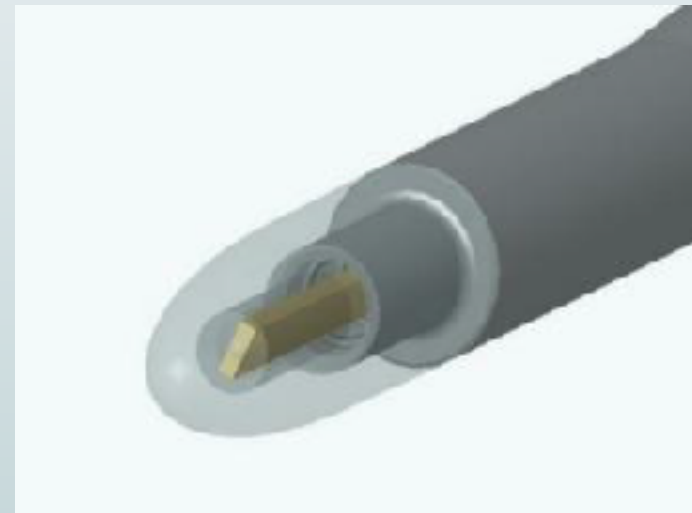
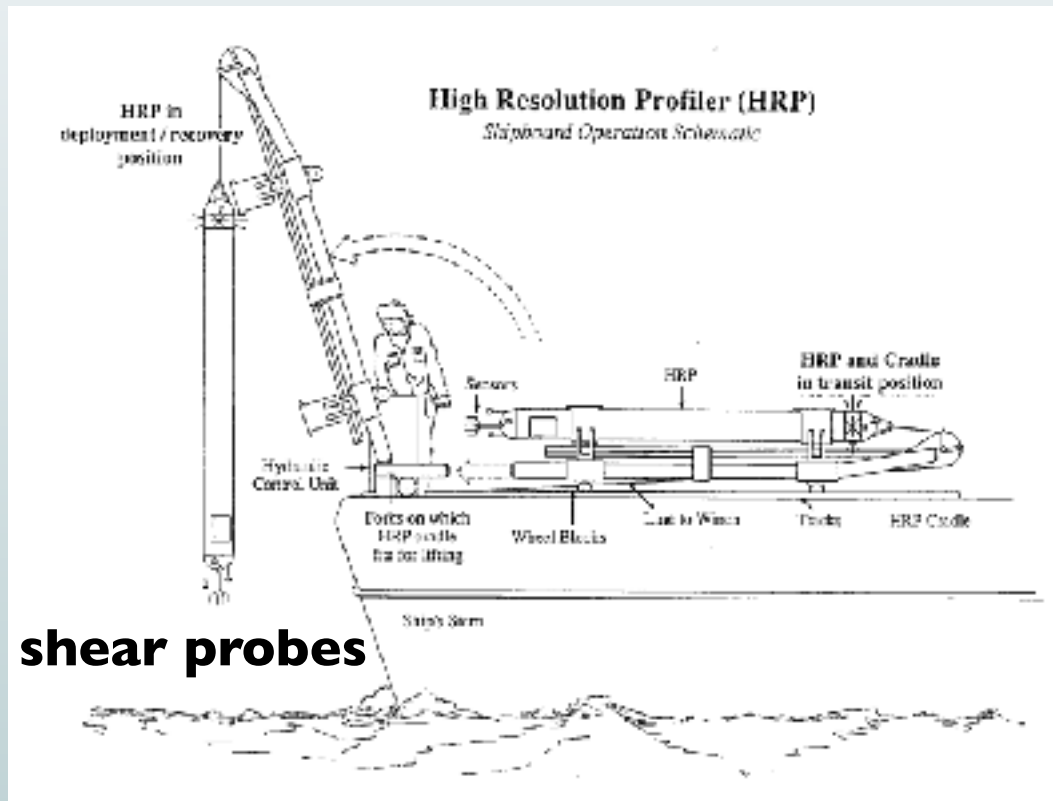
$$-\overline{\mathbf{u}'_h w'} \cdot \partial_z \bar{\mathbf{u}}_h + \overline{w' b'} \simeq \frac{15}{2} \nu \overline{(\partial_z u')^2}$$

- Introducing the flux Richardson number (*Osborn, 1981*)

$$Ri_f \equiv \frac{\overline{w' b'}}{\overline{\mathbf{u}'_h w'} \cdot \partial_z \bar{\mathbf{u}}_h} \quad \Longrightarrow \quad \overline{w' b'} \simeq -\frac{Ri_f}{1 - Ri_f} \frac{15}{2} \overline{(\partial_z u')^2} \equiv -\Gamma \epsilon$$

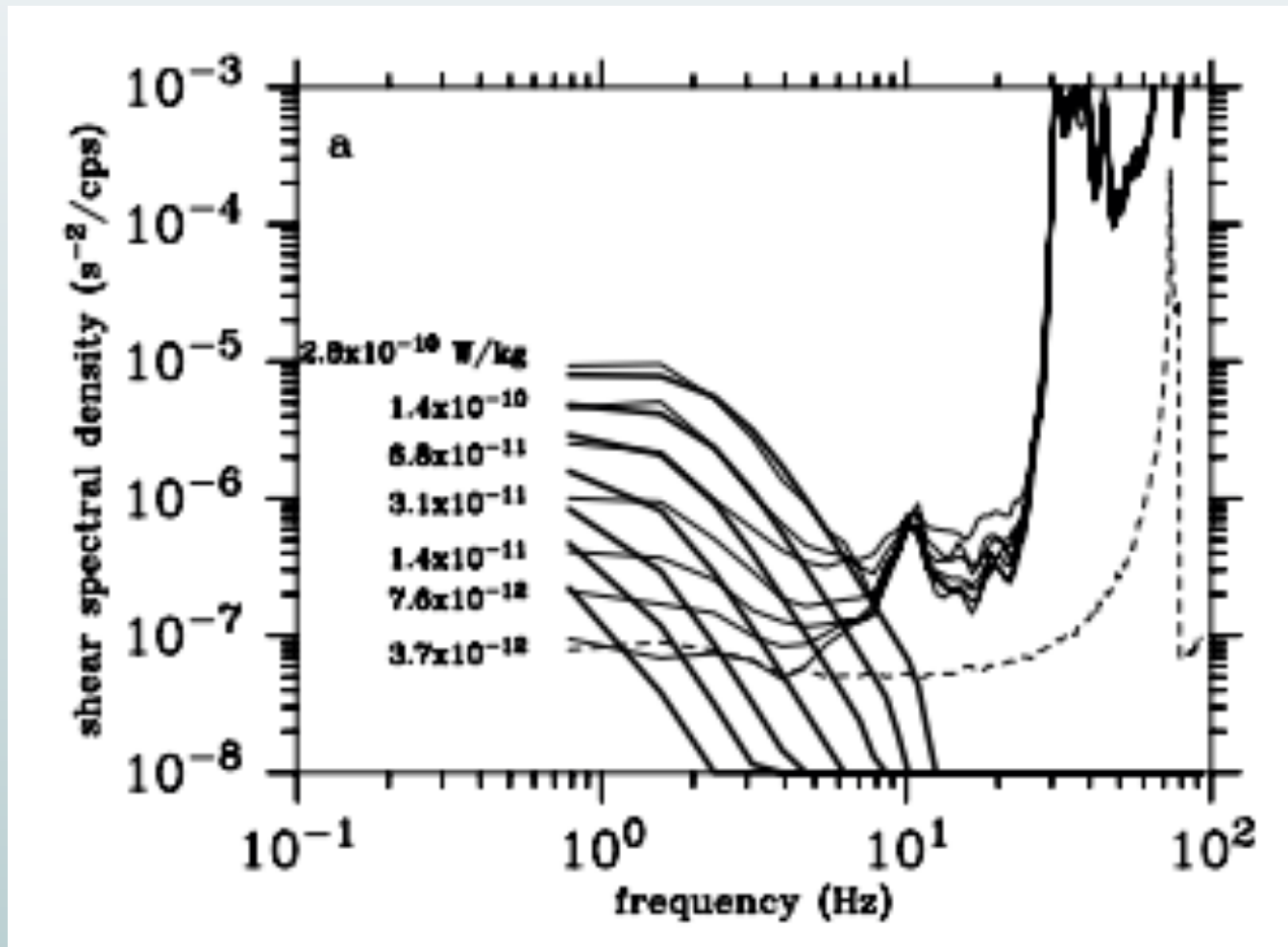
# Microstructure shear probes

## High Resolution Profiler



# Microstructure shear probes

Turbulent kinetic energy dissipation ( $\epsilon$ )



# Mixing coefficient

Temperature variance budget:  $\overline{w'b'} = -\frac{g\alpha}{2} \frac{\chi}{\partial_z \theta}$

Turbulent kinetic energy budget:  $\overline{w'b'} = -\Gamma\epsilon$

$$\implies \Gamma\epsilon = \frac{g\alpha}{2} \frac{\chi}{\partial_z \theta} \implies \Gamma = \frac{g\alpha}{2\partial_z \theta} \frac{\chi}{\epsilon} = 0.1 - 0.3$$

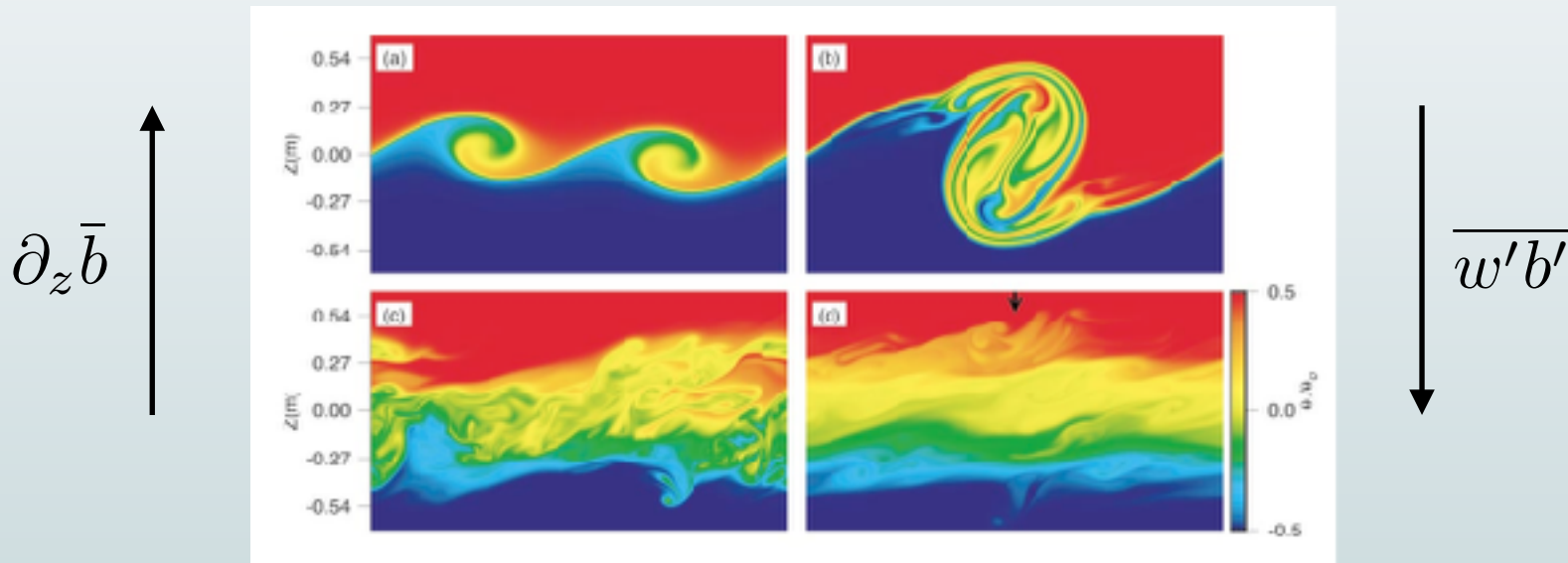
Table 4  $\chi/\epsilon$  in pycnoclines from direct  $\chi_T$  and  $\epsilon$  measurements, mostly by profiling

Location	$R_p$	$R_{\epsilon_b}$	$\Gamma$	Reference
Rockall Trough	>7	300–3,000	0.05–0.32	Oakey 1982, 1985 <sup>a</sup>
California Current	Variable	—	0.18 <sup>b</sup>	Gregg et al. 1986
Equator, 140°W <sup>c</sup>	Variable	1–10 <sup>6</sup>	0.12 <sup>b</sup>	Peters & Gregg 1988
Equator, 140°W <sup>c</sup>	—	—	0.12–0.48 <sup>d</sup>	Moum et al. 1989
California Current	—	—	0.05 <sup>e</sup>	Yamazaki & Osborn 1993
Admiralty Inlet	0.05	$\approx 2 \times 10^4$	0.58 <sup>f</sup>	Seim & Gregg 1994
Tidal channel	—	—	0.25, 0.23 <sup>g</sup>	Gargett & Moum 1995
Equator, 140°W <sup>h</sup>	—	20–10 <sup>5</sup>	0.14 <sup>h</sup>	Peters et al. 1995
Northeast Pacific	—	—	$\approx 0.3 - 0.4$	Moum 1996
Northeast Atlantic	–0.8 to 5	30–1,000	0.14, 0.21 <sup>i</sup>	Ruddick et al. 1997
Northeast Atlantic <sup>j</sup>	–100 to –1	—	0.16 $\pm$ 0.04	St. Laurent & Schmitt 1999
Monterey shelf <sup>k</sup>	>1	—	0.2–0.3	—
Equator, 80.5°E <sup>k</sup>	—	—	0.0022	Gregg & Horne 2000
Admiralty Inlet <sup>k</sup>	—	—	<0.02	Fujiwara et al. 2015

Gregg et al., 2018

# Eddy diffusivities

Diffusive closure:  $\overline{w'b'} = -K_T \partial_z \bar{b}$



Temperature variance budget

$$\overline{w'b'} = -\frac{g\alpha}{2} \frac{\chi}{\partial_z \bar{\theta}} \implies K_T^\chi = \frac{1}{2} \frac{\chi}{(\partial_z \bar{\theta})^2}$$

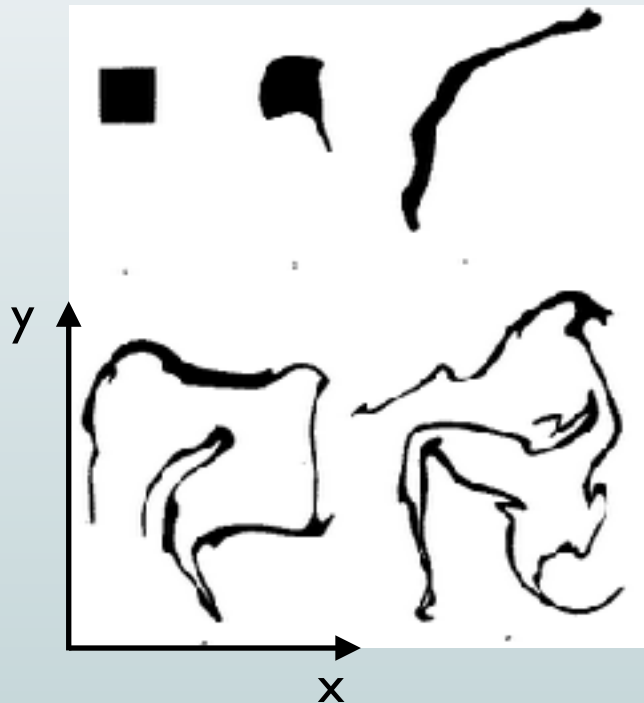
Turbulent kinetic energy budget

$$\overline{w'b'} = -\Gamma \epsilon \implies K_T^\epsilon = \Gamma \frac{\epsilon}{\partial_z \bar{b}}$$

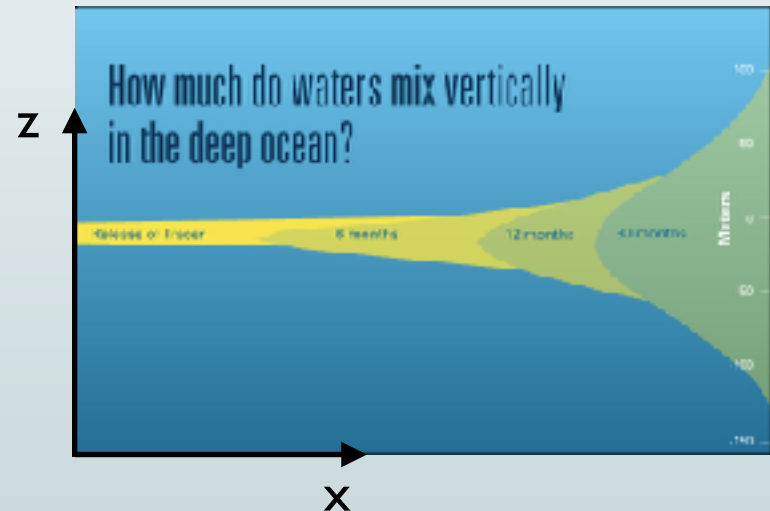


# IV. Tracer release experiments

Lateral stirring by  
geostrophic motions



Vertical mixing by breaking waves



$$\begin{aligned}\partial_t \bar{c} + \bar{\mathbf{u}} \cdot \nabla \bar{c} &= -\nabla \cdot \overline{\mathbf{u}'c'} \\ &= \nabla \cdot K_T \nabla \bar{c}\end{aligned}$$

# IV. Tracer release experiments

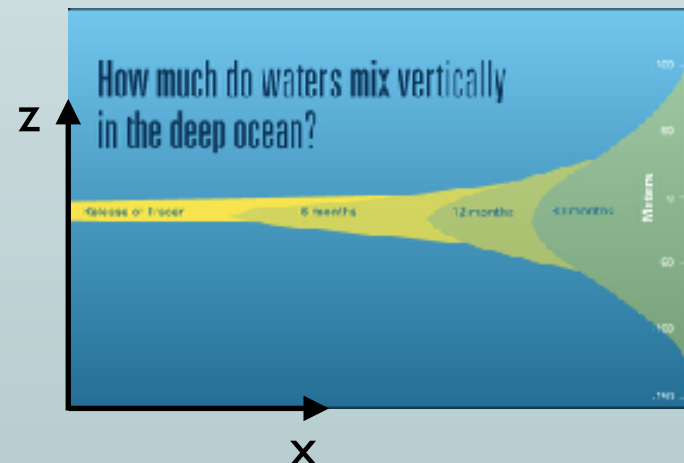
- The eddy diffusivity  $K_T$  is inferred from the evolution of the tracer  $z$ -moments

$$Z_1 \equiv \frac{\langle z c \rangle}{\langle c \rangle}, \quad Z_2 \equiv \frac{\langle (z - Z_1)^2 c \rangle}{\langle c \rangle}, \quad \langle \cdot \rangle = \iiint \cdot dV$$

- Assuming that
  - isopycnals are flat (can be relaxed using isopycnal coordinates)
  - the stirring velocity  $\bar{u}$  is along isopycnals ( $\bar{w}=0$ )
  - $K_T$  is independent of depth  $z$

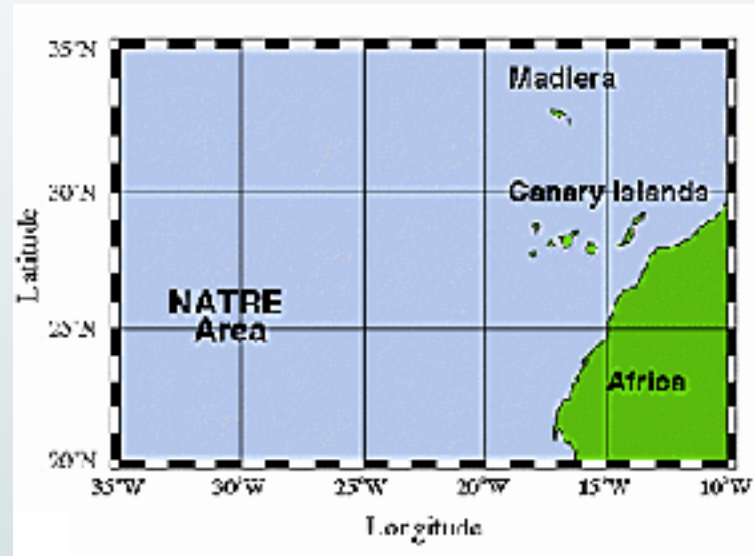
$$\frac{dZ_1}{dt} = 0$$

$$\frac{dZ_2}{dt} = 2 \frac{\langle K_T c \rangle}{\langle c \rangle}$$

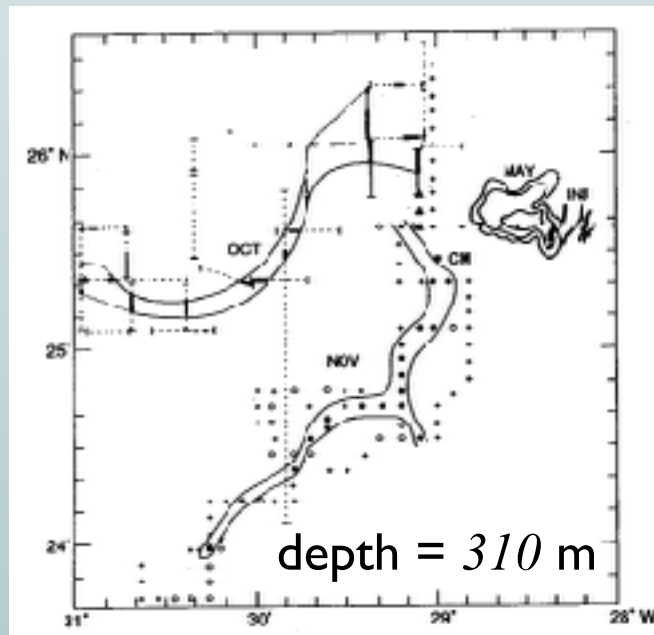


# North Atlantic Tracer Release Experiment (NATRE)

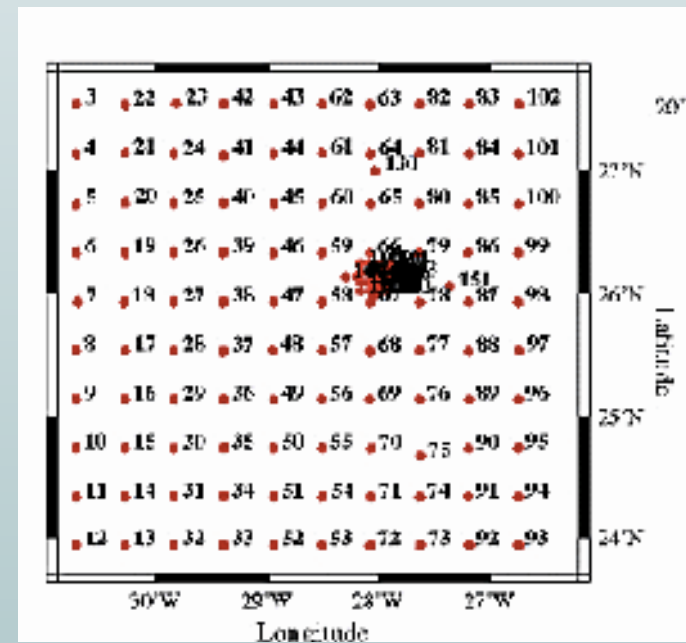
# NATRE



Tracer release (Ledwell et al., 1993)



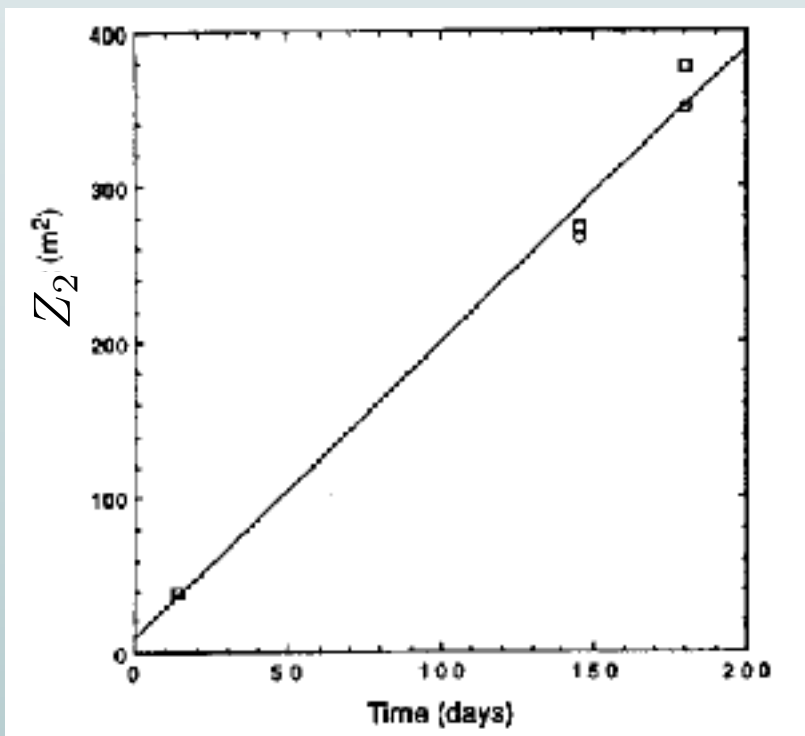
HRP profiles (Toole et al., 1994)



# Three methods are consistent

Tracer release (Ledwell et al., 1993)

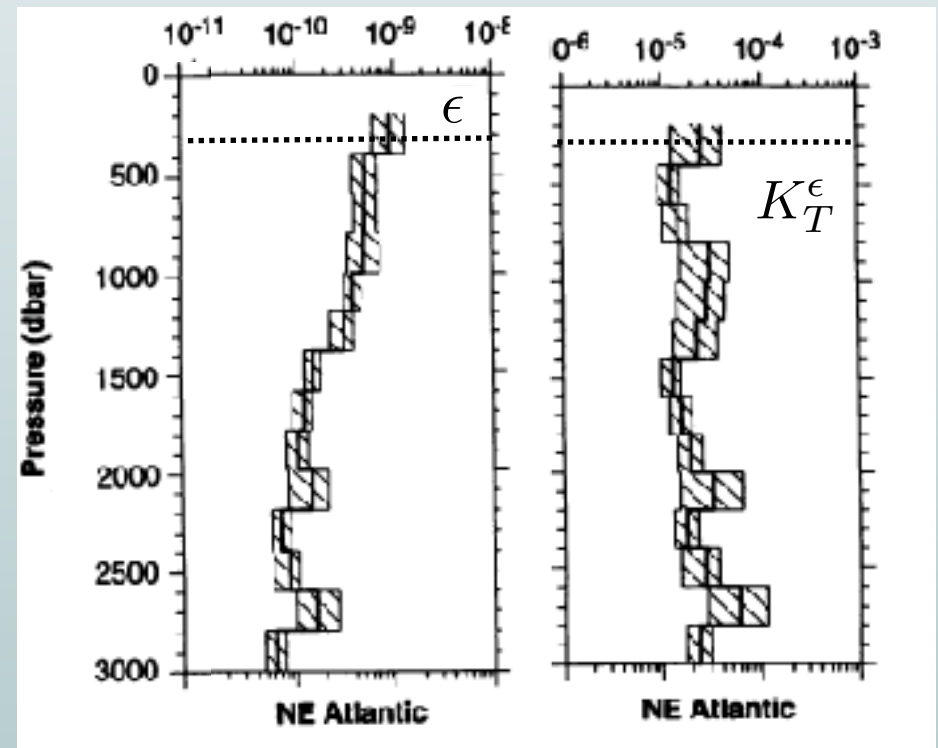
$$K_T^{tracer} = (1.1 \pm 0.2) \times 10^{-5} \text{m}^2 \text{s}^{-1}$$



HRP profiles (Toole et al., 1994)

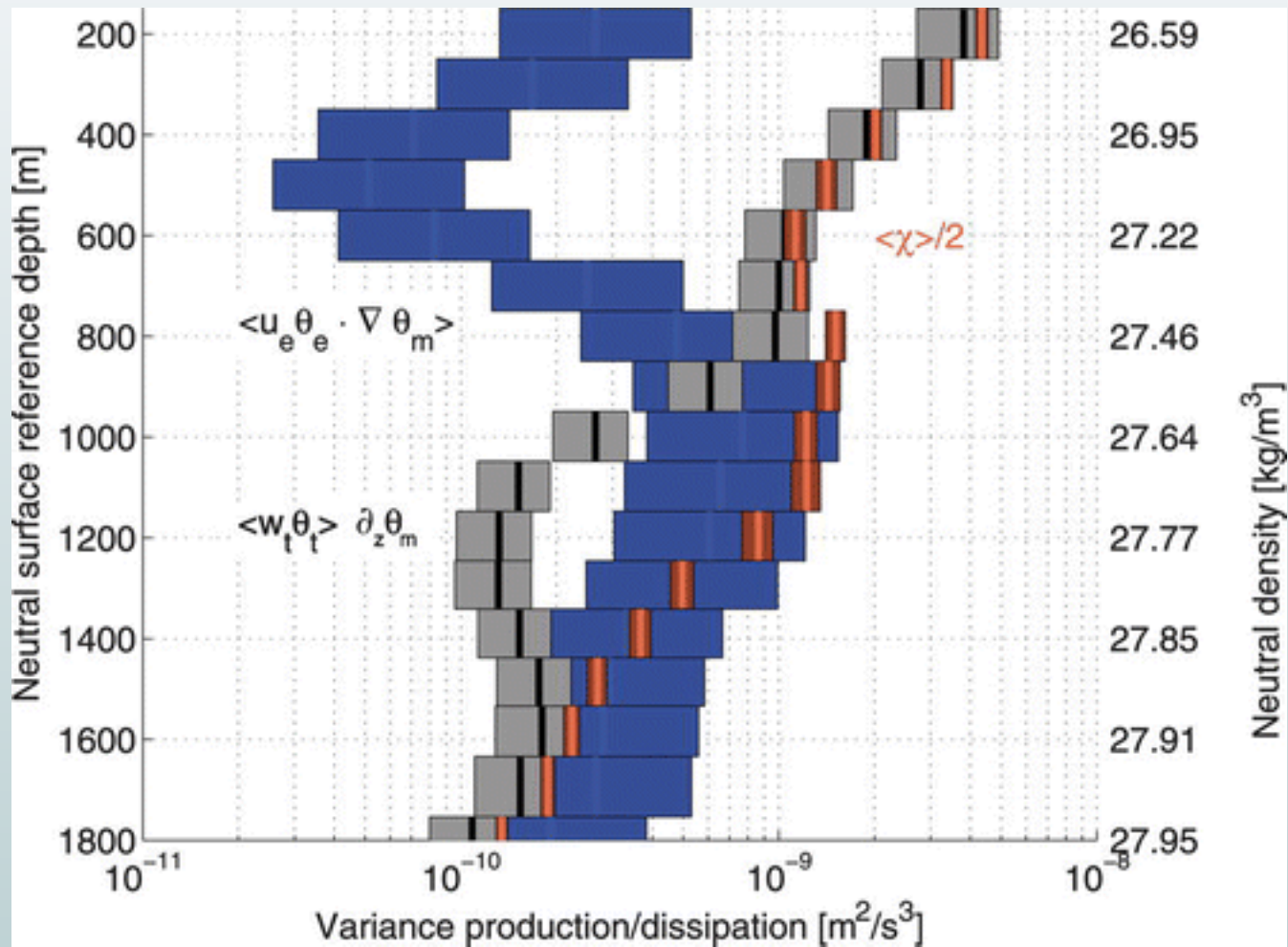
$$K_T^\chi = \frac{1}{2} \frac{\chi}{(\partial_z \theta)^2} = (1.0 \pm 0.2) \times 10^{-5} \text{m}^2 \text{s}^{-1}$$

$$K_T^\epsilon = 0.25 \frac{\epsilon}{\partial_z b} = (1.2 \pm 0.2) \times 10^{-5} \text{m}^2 \text{s}^{-1}$$





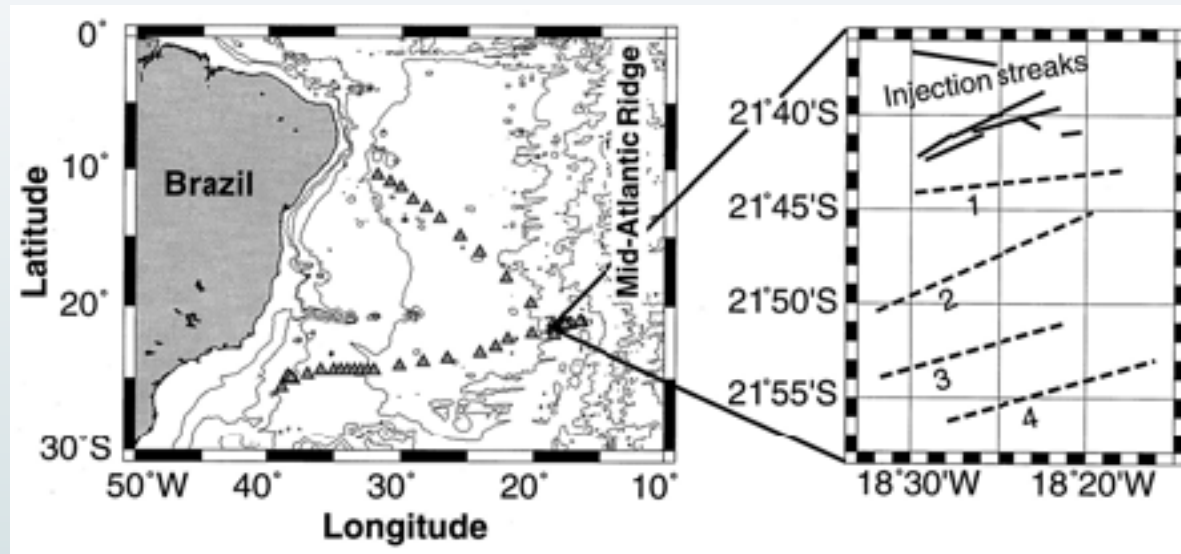
# Contamination of $\chi$



Ferrari and Polzin, 2005

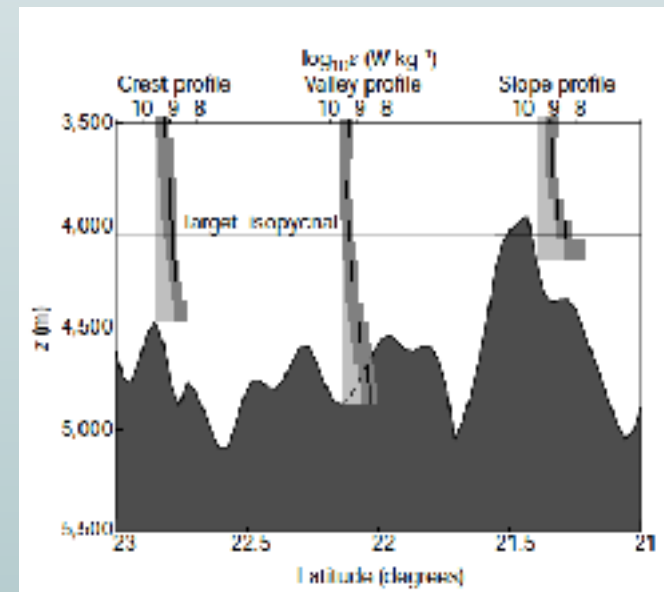
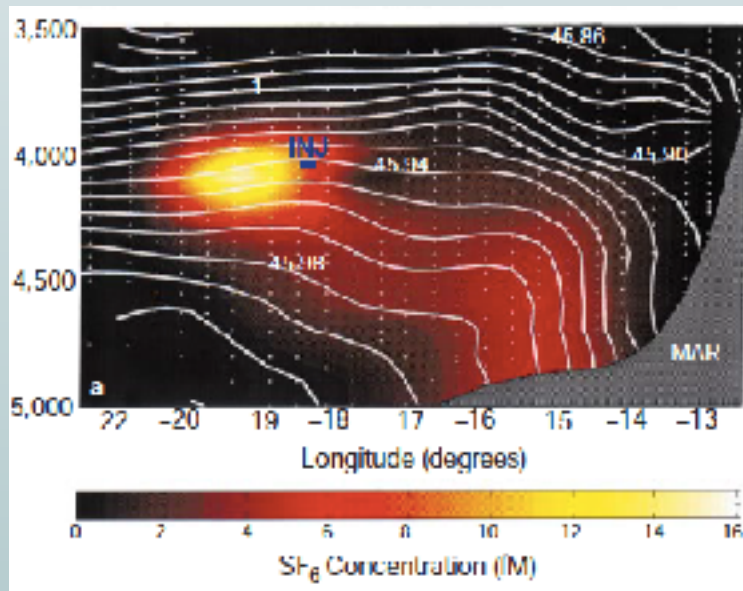
# Brazil Basin Tracer Release Experiment (BBTRE)

# BBTRE

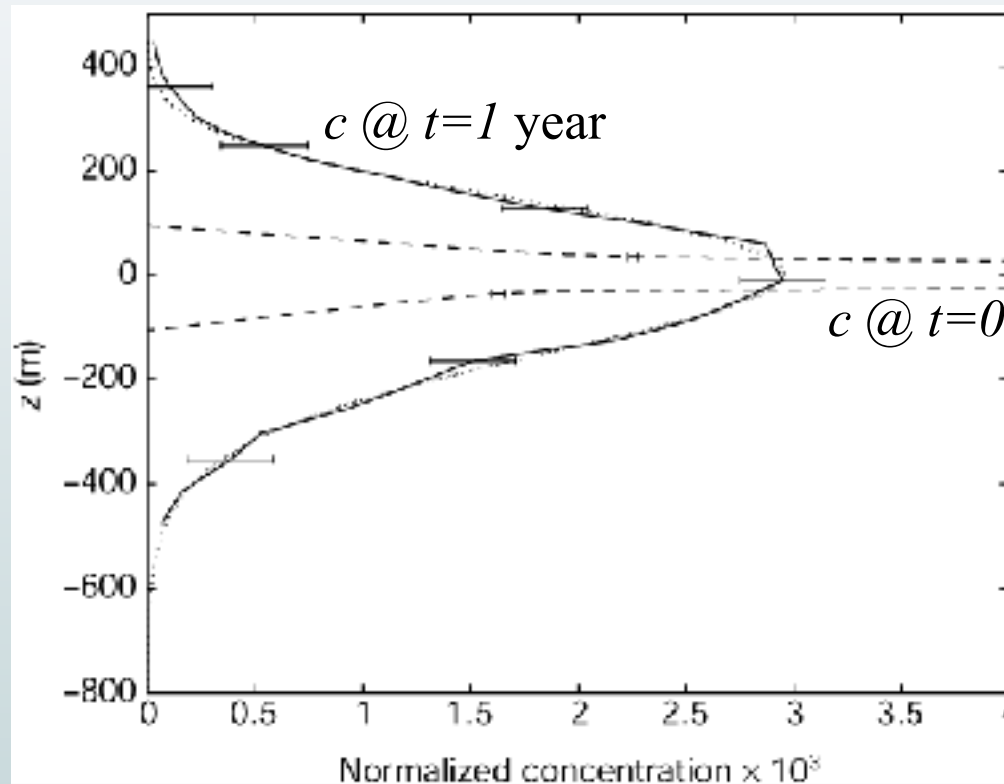


Tracer release (*Ledwell et al., 2000*)

HRP profiles (*Polzin et al., 1997*)



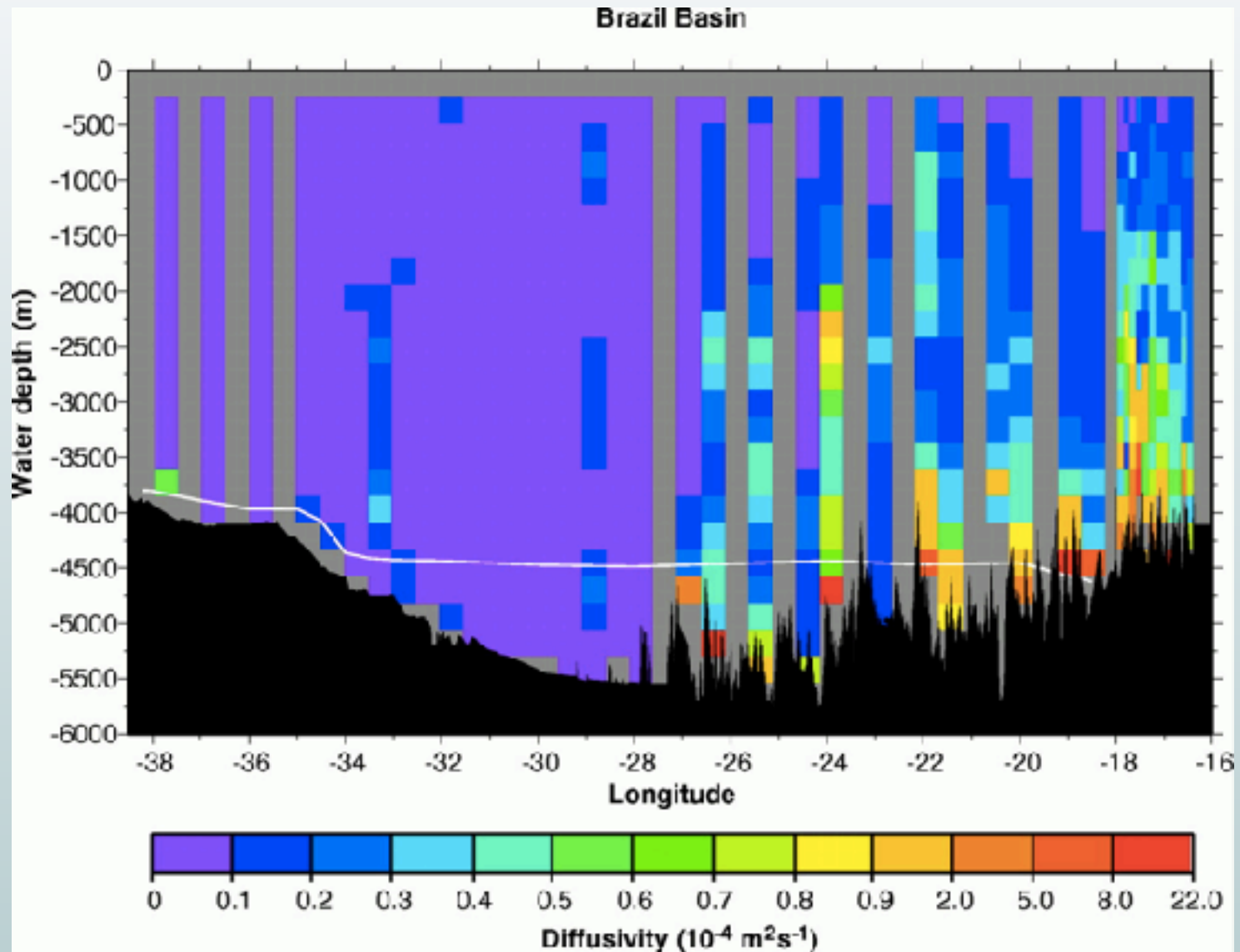
# $K_T$ is bottom enhanced



$$K_T^{tracer} = \frac{1}{2} \frac{Z_2|_{t=1 \text{ year}} - Z_2|_{t=0}}{1 \text{ year}} = (3 - 8) \times 10^{-4} \text{m}^2 \text{s}^{-1}$$

$$K_T^\epsilon = (2 \pm 1) \times 10^{-4} \text{m}^2 \text{s}^{-1}$$

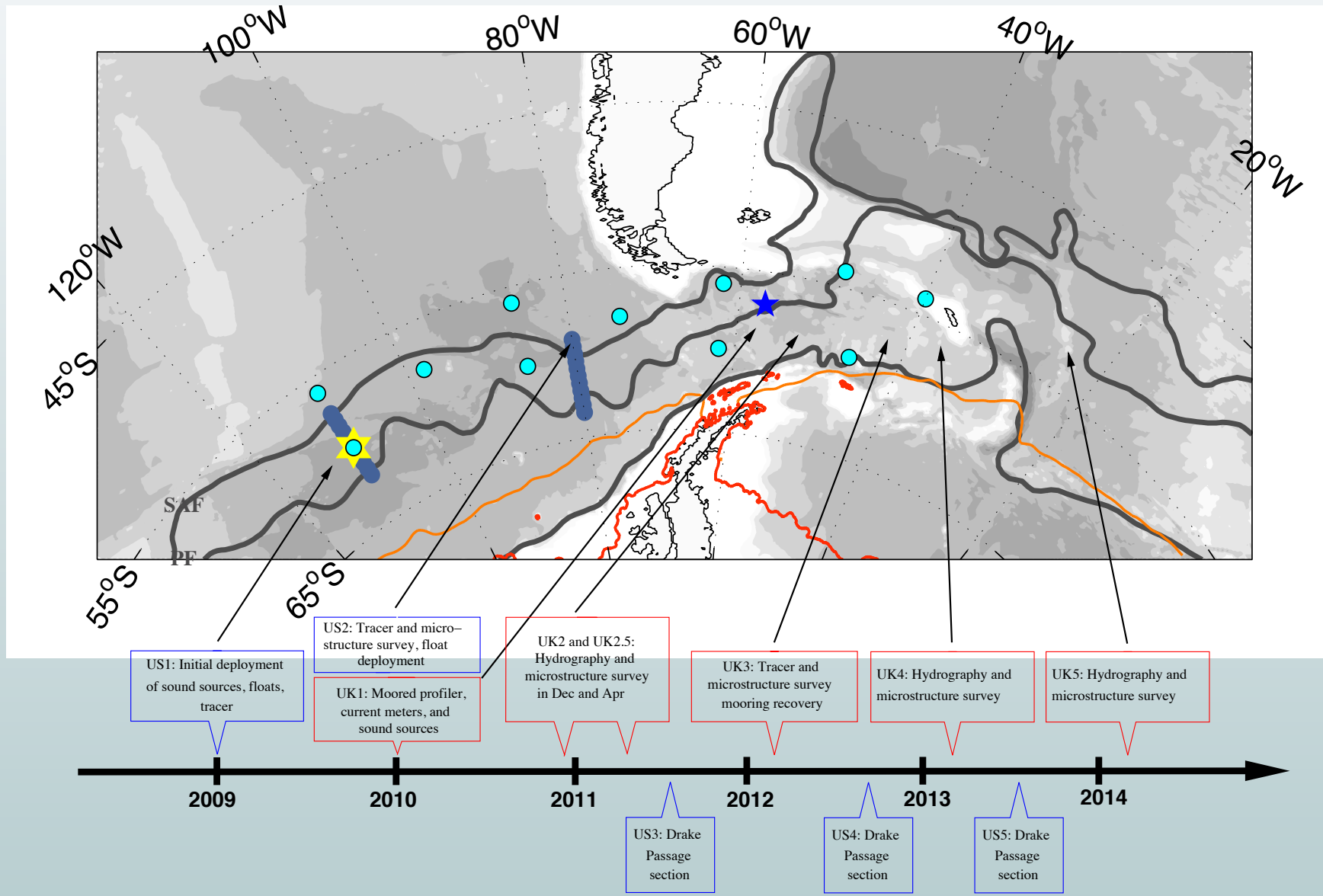
# $K_T$ is bottom enhanced



*Polzin et al., 1997*

Diapycnal and Isopycnal Mixing  
Experiment in the Southern  
Ocean (DIMES)

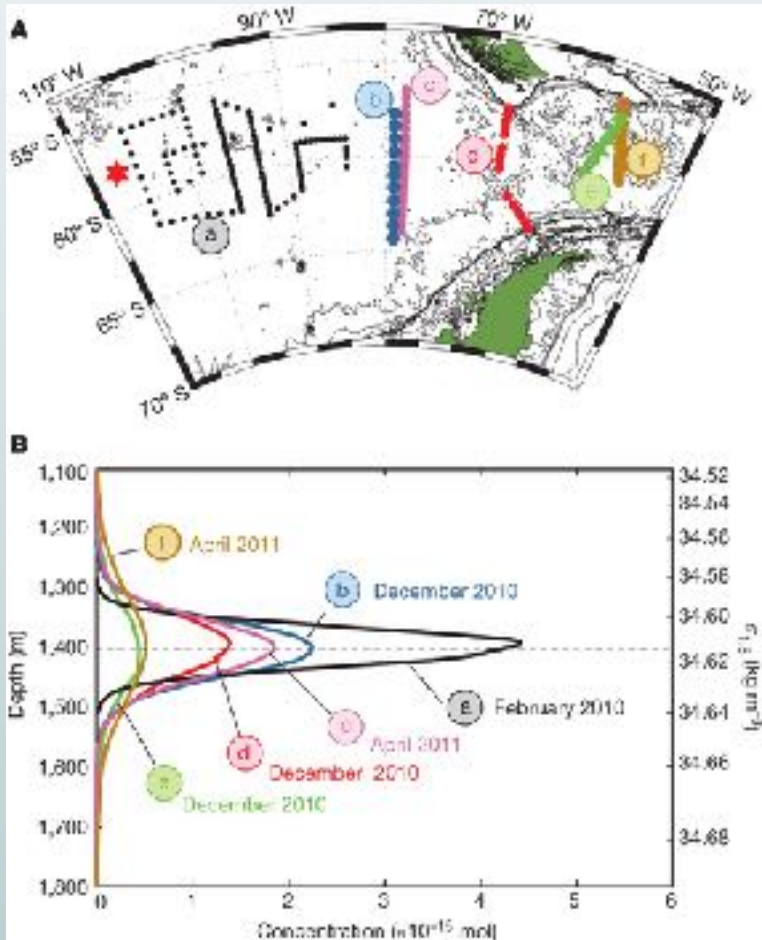
# DIMES





# Tracer estimate of $\kappa$

- Passive tracer ( $\text{CF}_3\text{SF}_5$ ) was released at 1500m depth, 2000m above seafloor



Upstream of Drake Passage  
between transects (a) and (d)

$$K_T^{tracer} = \frac{1}{2} \frac{Z_2|_b - Z_2|_*}{2 \text{ years}} \simeq 2 \times 10^{-5} \text{ m}^2 \text{ s}^{-1}$$

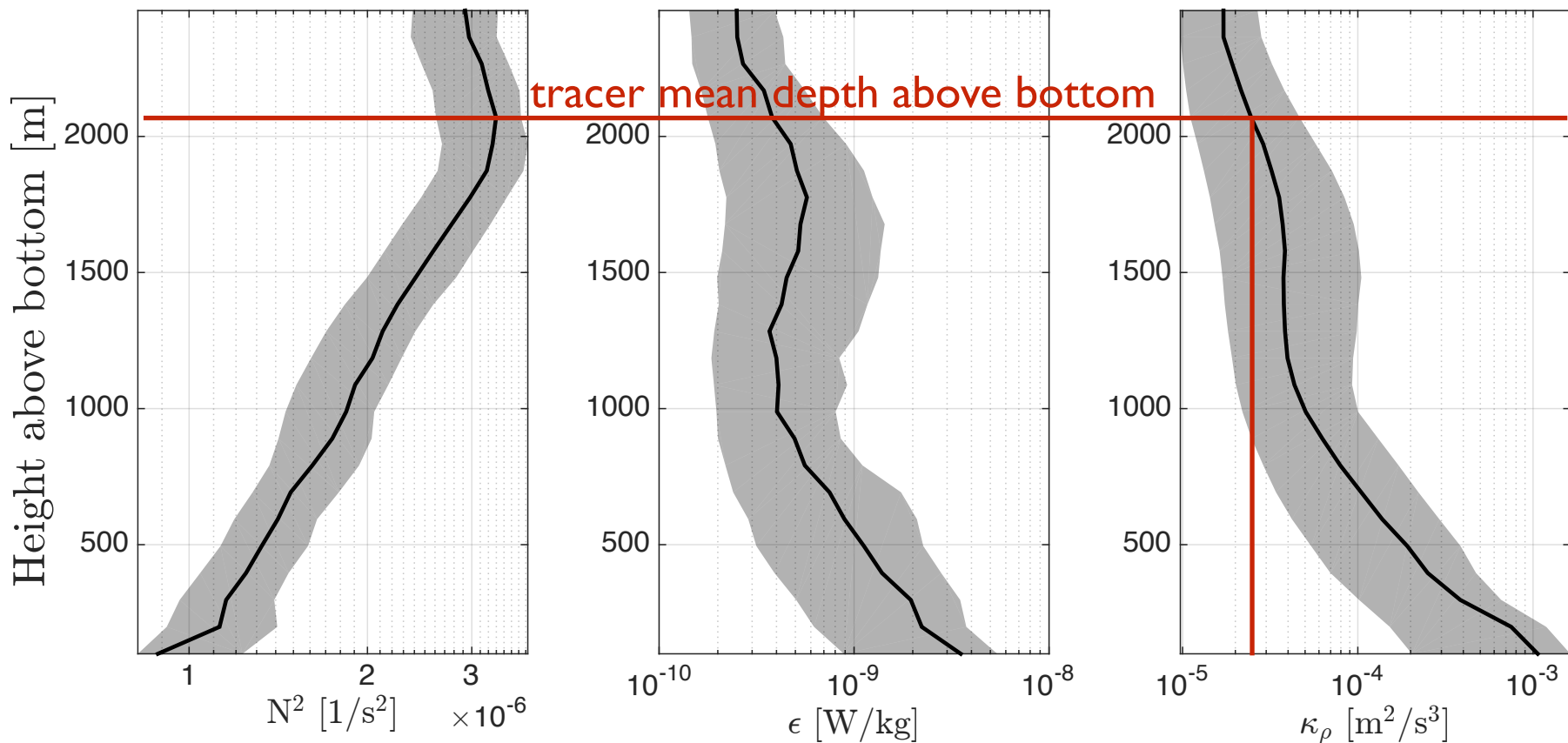
Downstream of Drake Passage  
between transects (d) and (f)

$$K_T^{tracer} = \frac{1}{2} \frac{Z_2|_f - Z_2|_d}{3 \text{ months}} \simeq 3 \times 10^{-4} \text{ m}^2 \text{ s}^{-1}$$



# Microstructure estimate of $K_T$

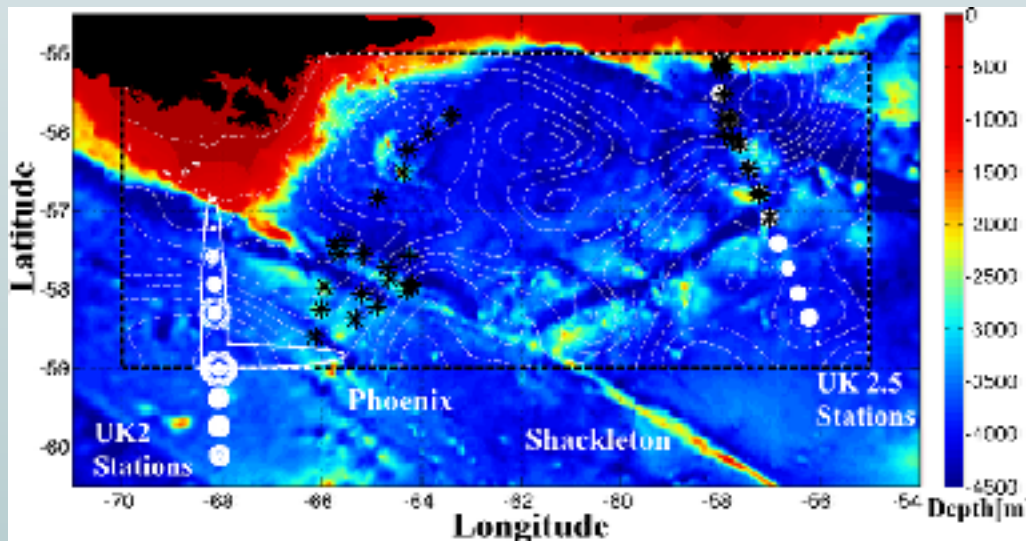
- Upstream of Drake Passage  $K_T^{tracer}$  and  $K_T^\epsilon$  are similar
- Downstream of Drake Passage  $K_T^\epsilon$  at mean depth of tracer is an order of magnitude smaller than  $K_T^{tracer}$



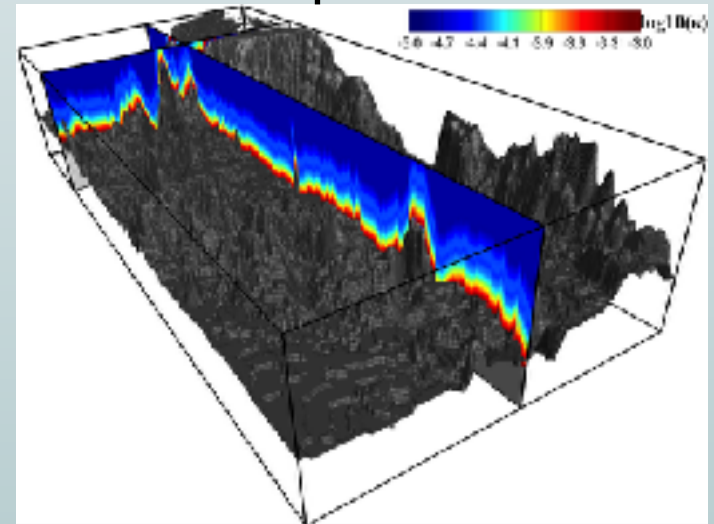
# Modeling approach

- MIT general circulation model of a 1400x400 km patch of Scotia Sea
- Resolution of 500-600 m, 100 vertical levels (30 m resolution at tracer depth)
- Model forced at boundaries with a coarser resolution patch of ACC constrained to observations (Tulloch, Ferrari et al., 2014)
- Smith and Sandwell topography at one minute (1/60th of degree)
- Vertical profile of  $K_T$  from microstructure profile imposed everywhere as a function of height above the bottom

Model domain

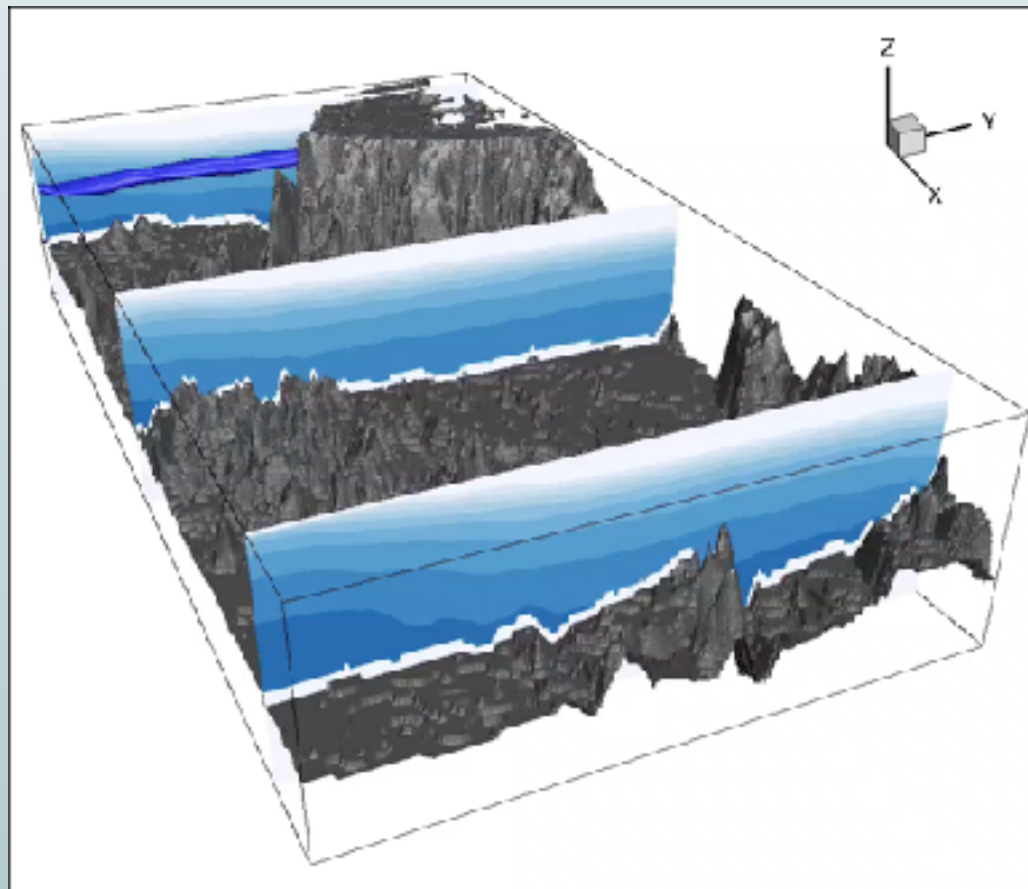


$K_m$  profiles



# Tracer evolution

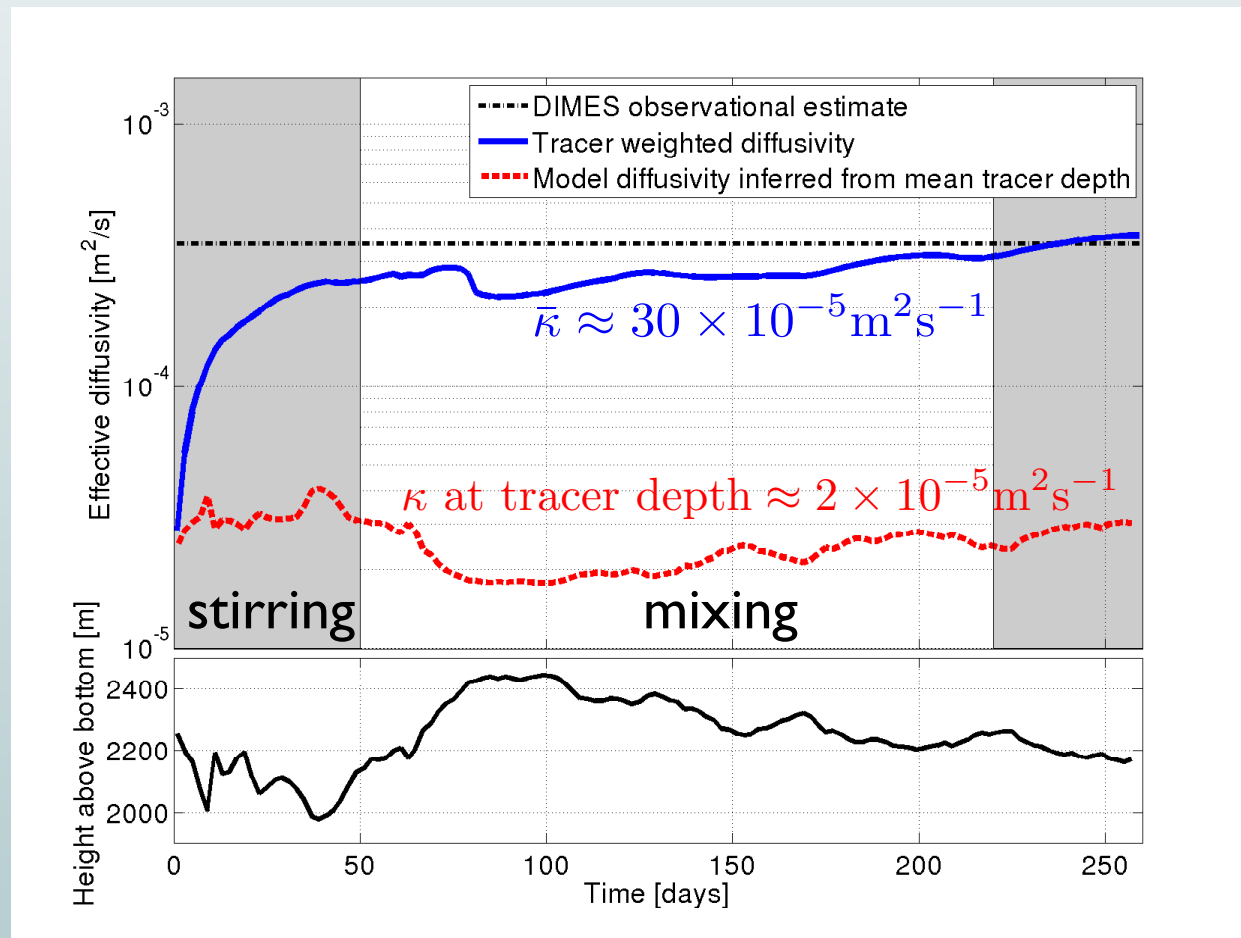
- Tracer is released along western boundary of domain based on sampling of real tracer along that line
- Movie shows that tracer is
  - stirred laterally by geostrophic eddies toward seamounts/ridges
  - mixed vertically by enhanced  $K_T$  next to seamounts/ridges



# Numerical estimate of $K_T^{tracer}$

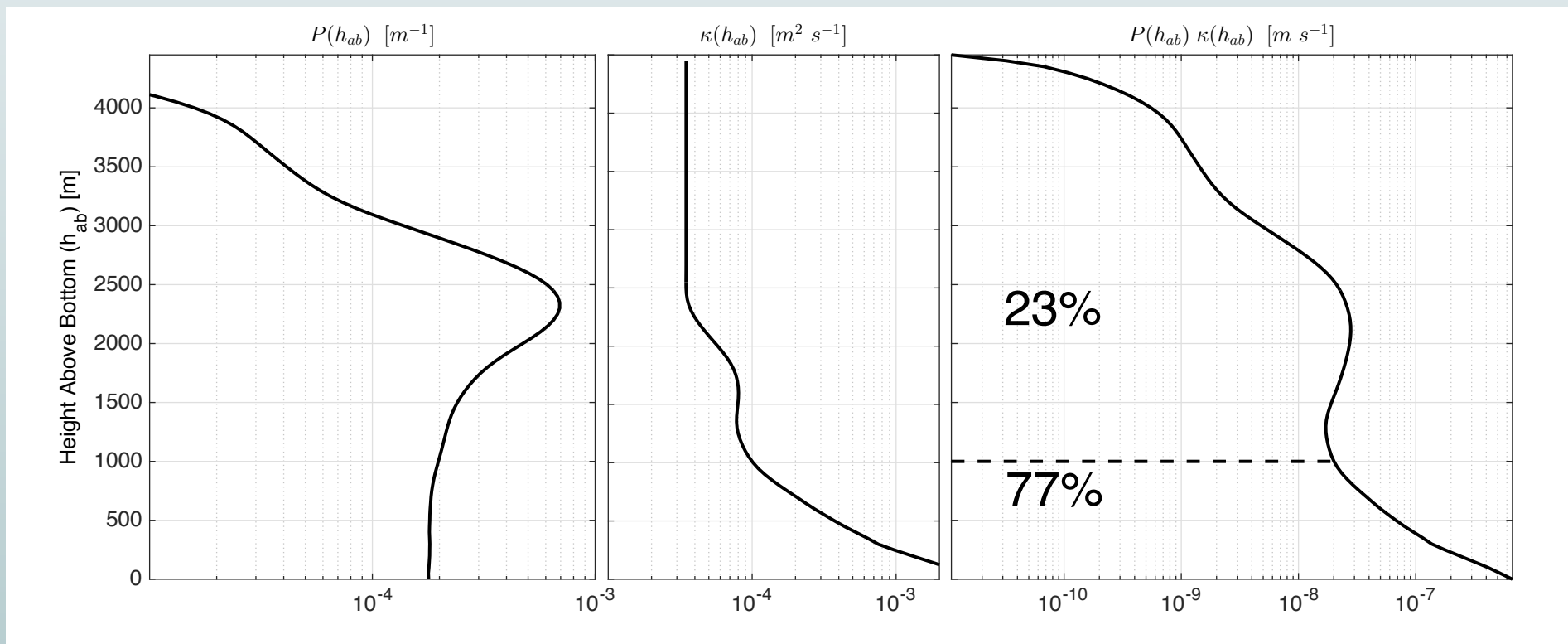
- We can estimate the diffusivity experienced by the tracer as

$$\bar{K} \equiv \frac{\iiint K_T c \, dV}{\iiint c \, dV}$$



# Numerical estimate of $K_T^{tracer}$

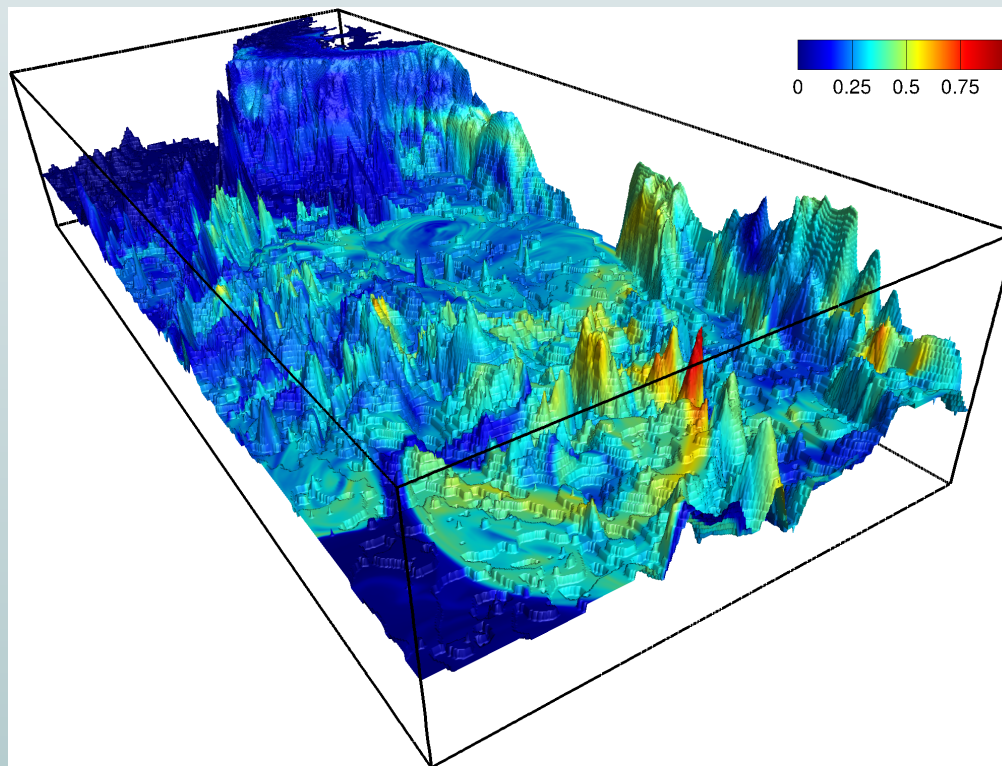
- Panel 1: distribution of tracer peaks  $\sim 2300$  m above seafloor and decays below
- Panel 2: profile of  $K_T$  increases toward seafloor
- Panel 3: increase of  $K_T$  is so large that estimate of  $\bar{K}$  is dominated by integral over bottom 1000 m
- **Mixing occurs when tracer is within 1000 m of seafloor**



# Numerical estimate of $K_T^{tracer}$

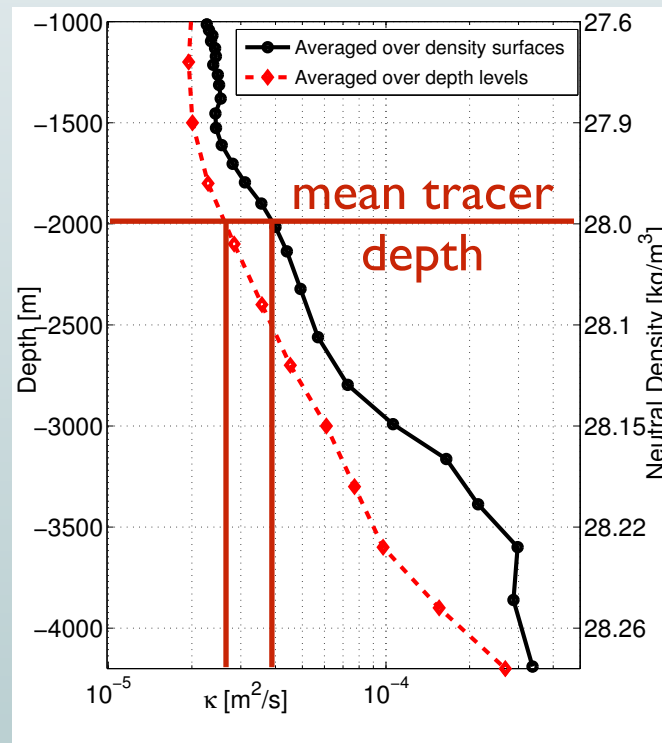
- Tracer tends to accumulate over remounts and ridges where mixing is strong
  - velocities are weaker close to seafloor
  - bottom enhanced mixing drives tracers toward the seafloor
  - other reasons?

Vertically integrated tracer concentration



# Numerical estimate of $K_T^{tracer}$

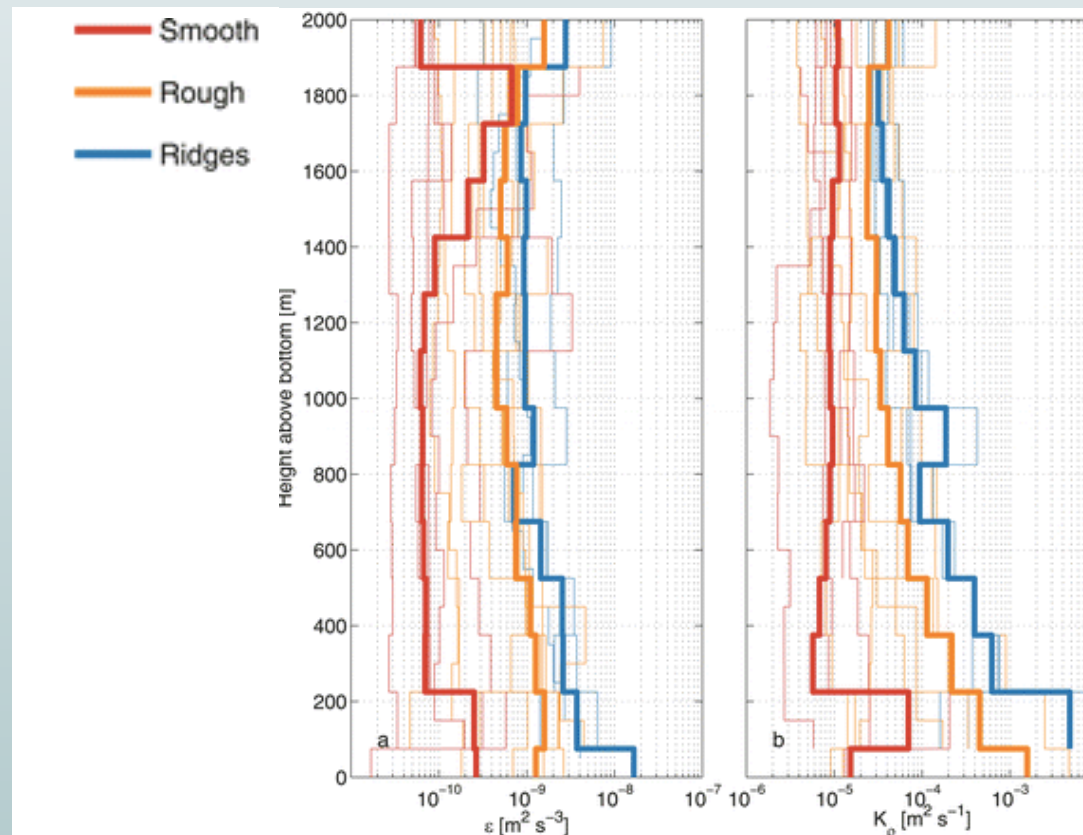
- Simply averaging along density surfaces gives little enhancement (mixing hotspots are too rare)
- Accumulation of tracer near topography is crucial to explain large  $K_T^{tracer}$  in deep ocean (hotspots trap tracer)





# Conclusions

- ▶  $K_T^\epsilon = \Gamma\epsilon / \partial_z \bar{b}$  increases toward rough seafloor
- ▶  $\overline{w'b'} = -\Gamma\epsilon$  increases in magnitude toward rough seafloor
- ▶ mixing is confined within a few hundred meters of seafloor





# Conundrum

- ▶  $\overline{w'b'} = -\Gamma\epsilon$  increases in magnitude toward rough seafloor
- ▶ mixing drives sinking, not upwelling of ocean waters

