Deep Ocean Mixing

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Turbulence and diapycnal mixing

Methods used to estimate ocean diapycnal mixing
- temperature variance budget
- turbulent kinetic energy budget
- tracer release experiments

Direct estimates of ocean diapycnal mixing
- North Atlantic (shallow)
- Brazil Basin (deep)
- Drake Passage (mid-depth)
Turbulence and mixing
Turbulence in ocean interior

Subinertial motions have large $Ri$ and small $Ro$

Superinertial motions can develop small $Ri$ and large $Ro$
Breaking internal waves

Depth (10 m)

Light water

Dense water
Turbulent buoyancy fluxes
Boussinesq Equations

\[\begin{align*}
\partial_t u + (u \cdot \nabla) u + f \hat{z} \times u &= -\rho_0^{-1} \nabla p + b z + \nabla \cdot (\nu \nabla u), \\
\nabla \cdot u &= 0, \\
\partial_t \theta + (u \cdot \nabla) \theta &= \nabla \cdot (\kappa_\theta \nabla \theta), \\
\partial_t S + (u \cdot \nabla) S &= \nabla \cdot (\kappa_S \nabla S), \\
b &= g\alpha(\theta - \theta_0) - g\beta(S - S_0), \\
\rho &= \rho_0 \left(1 - g^{-1}b\right)
\end{align*}\]

\[\Rightarrow \quad \partial_t b + (u \cdot \nabla) b = \nabla \cdot \left(\kappa_\theta g\alpha \nabla \theta - \kappa_S g\beta \nabla S\right)\]
Reynolds decomposition

Buoyancy budget

$$\partial_t \bar{b} + (\bar{u} \cdot \nabla) \bar{b} = \nabla \cdot (\kappa_{\theta} g \alpha \nabla \theta - \kappa_S g \beta \nabla \bar{S})$$

Subinertial buoyancy budget

$$\partial_t \bar{b} + (\bar{u} \cdot \nabla) \bar{b} = - \nabla \cdot (\bar{u}'b' - \kappa_{\theta} g \alpha \nabla \bar{\theta} + \kappa_S g \beta \nabla \bar{S})$$

$$\simeq - \nabla \cdot \bar{u}'b'$$

$$\simeq - \partial_z \bar{w}'b'$$
Measurements of turbulent buoyancy fluxes
I. Direct eddy correlations

- Compute $w$ and $b$ ($\theta$ and $S$)
- High pass signals to extract super-inertial signals $w'$ and $b'$
- Compute correlations by averaging over time $\overline{w'b'}$

Fleury and Lueck, 1994

- Challenge I: measure $w$ and $b$ ($\theta$ and $S$) at same location from stable platform
- Challenge II: average over long enough time
II. Temperature variance budget

- Potential temperature budget

\[ \frac{\partial_t \bar{\theta}}{} + (\mathbf{u} \cdot \nabla) \bar{\theta} = \nabla \cdot (\kappa_{\theta} \nabla \bar{\theta}) \]

- Mean and eddy potential temperature budget

\[ \frac{\partial_t \bar{\theta}}{} + (\bar{\mathbf{u}} \cdot \nabla) \bar{\theta} = \nabla \cdot (-\bar{\mathbf{u}'\theta'} + \kappa_{\theta} \nabla \bar{\theta}) \]
\[ \frac{\partial_t \theta'}{} + ((\bar{\mathbf{u}} + \mathbf{u}') \cdot \nabla) \theta' + \mathbf{u}' \cdot \nabla \bar{\theta} = \nabla \cdot (\mathbf{u}'\theta' + \kappa_{\theta} \nabla \theta') \]

- Potential temperature variance budget

\[ \frac{1}{2} \frac{\partial_t \bar{\theta}'}{} + \frac{1}{2} \nabla \cdot \left[ (\bar{\mathbf{u}} + \mathbf{u}') \theta'^2 - \kappa_{\theta} \nabla \theta'^2 \right] = -\mathbf{u}'\theta' \cdot \nabla \bar{\theta} - \kappa_{\theta} |\nabla \theta'|^2 \]
II. Temperature variance budget

- Assuming that turbulence in stationary, homogeneous and isotropic:

\[
\frac{1}{2} \frac{\partial}{\partial t} \overline{\theta'^2} + \frac{1}{2} \nabla \cdot \left[ (\overline{u + u'}) \theta'^2 - \kappa_\theta \nabla \theta'^2 \right] = -\overline{u' \theta'} \cdot \nabla \overline{\theta} - \kappa_\theta |\nabla \theta'|^2 - 3 \kappa_\theta (\partial_z \theta')^2
\]

stationarity                homogeneity                                        isotropy

- Assuming that \( \nabla \overline{\theta} \simeq \partial_z \overline{\theta} \hat{z} \)

\[
\overline{w' \theta'} \simeq -\frac{3 \kappa_\theta (\partial_z \theta')^2}{\partial_z \overline{\theta}} \equiv -\frac{1}{2} \frac{\chi}{\partial_z \overline{\theta}}
\]

- Assuming that temperature dominates buoyancy gradients

\[
\overline{w'b'} \simeq g \alpha \overline{w' \theta'} \simeq -\frac{g \alpha}{2} \frac{\chi}{\partial_z \overline{\theta}}
\]  
(Osborn-Cox, 1972)
Microstructure thermistors

High Resolution Profiler (HRP)
Temperature gradient spectra ($\chi$)
III. Kinetic energy budget

- Full turbulent kinetic energy budget
  \[
  \frac{1}{2} \frac{\partial_t |\mathbf{u'}|^2}{2} + \nabla \cdot \left[ \left( \overline{\mathbf{u} + \mathbf{u'}} \right) |\mathbf{u'}|^2 - \nu \nabla |\mathbf{u'}|^2 + \rho_0^{-1} p' w' \right] = \\
  = -\overline{\mathbf{u'} \mathbf{u'}} \cdot \nabla \overline{\mathbf{u}} + \overline{w' b'} - \nu |\nabla \mathbf{u'}|^2
  \]

- Turbulent kinetic energy budget for stationary, homogeneous, isotropic turbulence
  \[
  -\overline{\mathbf{u'} \mathbf{u'}} \cdot \nabla \overline{\mathbf{u}} + \overline{w' b'} \simeq \frac{15}{2} \nu (\partial_z \mathbf{u'})^2
  \]

- Assuming that \( \nabla \overline{\mathbf{u}} \simeq \partial_z \overline{\mathbf{u}_h} \hat{z} \)
  \[
  -\overline{\mathbf{u}_h' w'} \cdot \partial_z \overline{\mathbf{u}_h} + \overline{w' b'} \simeq \frac{15}{2} \nu (\partial_z \mathbf{u'})^2
  \]

- Introducing the flux Richardson number (Osborn, 1981)
  \[
  Ri_f \equiv \frac{\overline{w' b'}}{\overline{u_h' w'} \cdot \partial_z \overline{u_h}} \quad \Rightarrow \quad \overline{w' b'} \simeq -\frac{Ri_f}{1 - Ri_f} \frac{15}{2} (\partial_z \mathbf{u'})^2 \equiv -\Gamma \epsilon
  \]
Microstructure shear probes

High Resolution Profiler

shear probes
Microstructure shear probes

Turbulent kinetic energy dissipation ($\epsilon$)
Mixing coefficient

Temperature variance budget: \( \overline{w'b'} = -\frac{g\alpha}{2} \frac{\chi}{\partial_z \bar{\theta}} \)

Turbulent kinetic energy budget: \( \overline{w'b'} = -\Gamma \epsilon \)

\[ \Rightarrow \Gamma \epsilon = \frac{g\alpha}{2} \frac{\chi}{\partial_z \bar{\theta}} = 0.1 - 0.3 \]
Eddy diffusivities

Diffusive closure: \( \overline{w'b'} = -K_T \partial_z \overline{b} \)

Temperature variance budget

\[
\overline{w'b'} = -\frac{g \alpha}{2} \frac{\chi}{\partial_z \theta} \quad \implies \quad K_T^\chi = \frac{1}{2} \frac{\chi}{(\partial_z \theta)^2}
\]

Turbulent kinetic energy budget

\[
\overline{w'b'} = -\Gamma \epsilon \quad \implies \quad K_T^\epsilon = \Gamma \frac{\epsilon}{\partial_z \overline{b}}
\]
IV. Tracer release experiments

Lateral stirring by geostrophic motions

Vertical mixing by breaking waves

\[
\partial_t \bar{c} + \bar{u} \cdot \nabla \bar{c} = -\nabla \cdot \bar{u}' \bar{c}' = \nabla \cdot K_T \nabla \bar{c}
\]
IV. Tracer release experiments

- The eddy diffusivity $K_T$ is inferred from the evolution of the tracer $z$-moments

\[
Z_1 \equiv \frac{\langle z c \rangle}{\langle c \rangle}, \quad Z_2 \equiv \frac{\langle (z - Z_1)^2 c \rangle}{\langle c \rangle}, \quad \langle \cdot \rangle = \iiint \cdot \, dV
\]

- Assuming that
  - isopycnals are flat (can be relaxed using isopycnal coordinates)
  - the stirring velocity $\bar{u}$ is along isopycnals ($\bar{w}=0$)
  - $K_T$ is independent of depth $z$

\[
\frac{dZ_1}{dt} = 0, \quad \frac{dZ_2}{dt} = 2 \frac{\langle K_T c \rangle}{\langle c \rangle}
\]
North Atlantic Tracer Release Experiment (NATRE)
NATRE

Tracer release (Ledwell et al., 1993)

HRP profiles (Toole et al., 1994)

depth = 310 m
Three methods are consistent

**Tracer release (Ledwell et al., 1993)**

\[ K_{T}^{\text{tracer}} = (1.1 \pm 0.2) \times 10^{-5} \text{ m}^2 \text{ s}^{-1} \]

**HRP profiles (Toole et al., 1994)**

\[ K_{T}^{\chi} = \frac{1}{2} \frac{\chi}{(\partial_z \theta)^2} = (1.0 \pm 0.2) \times 10^{-5} \text{ m}^2 \text{ s}^{-1} \]
\[ K_{T}^{\epsilon} = 0.25 \frac{\epsilon}{\partial_z b} = (1.2 \pm 0.2) \times 10^{-5} \text{ m}^2 \text{ s}^{-1} \]
Contamination of $\chi$

Ferrari and Polzin, 2005
Brazil Basin Tracer Release Experiment (BBTRE)
Tracer release (*Ledwell et al., 2000*)

HRP profiles (*Polzin et al., 1997*)
$K_T$ is bottom enhanced

\[ K_{T}^{tracer} = \frac{1}{2} \frac{Z_2|t=1\text{ year} - Z_2|t=0}{1\text{ year}} = (3 - 8) \times 10^{-4} \text{ m}^2\text{s}^{-1} \]

\[ K_{T}^{e} = (2 \pm 1) \times 10^{-4} \text{ m}^2\text{s}^{-1} \]

Ledwell et al., 2000
$K_T$ is bottom enhanced

Polzin et al., 1997
Diapycnal and Isopycnal Mixing Experiment in the Southern Ocean (DIMES)
This is the final data report of acoustically tracked 'shallow deployment' RAFOS float data collected during the 2010-2012 "Critical Layers and Isopycnal Mixing in the Southern Ocean" (hereafter CLIMS) and "Diapycnal and Isopycnal Mixing Experiment in the Southern Ocean" (hereafter DIMES).

DIMES was designed to measure eddy mixing along (isopycnal) and across (diapycnal) density surfaces in the subsurface ocean, and to determine how those processes depend on the larger scale dynamics of the ocean (Figure 1).

The resultant horizontal resolution of mixing rates will subsequently allow for better representation of such processes in numerical models of ocean circulation and climate.

Figure 1: Location and timeline of past and future DIMES cruises. The turquoise circles indicate approximate location of sound sources.

CLIMS was designed as a collaborative project to supply additional floats, to enhance vertical resolution of isopycnal mixing rates given present evidence for its vertical variability (Klocker et al. (2012)).
Passive tracer (CF$_3$SF$_5$) was released at 1500m depth, 2000m above seafloor

- Upstream of Drake Passage between transects (a) and (d)

\[
K_{T}^{\text{tracer}} = \frac{1}{2} \frac{Z_2|_b - Z_2|_\star}{2 \text{ years}} \simeq 2 \times 10^{-5} \text{ m}^2\text{s}^{-1}
\]

- Downstream of Drake Passage between transects (d) and (f)

\[
K_{T}^{\text{tracer}} = \frac{1}{2} \frac{Z_2|_f - Z_2|_d}{3 \text{ months}} \simeq 3 \times 10^{-4} \text{ m}^2\text{s}^{-1}
\]
Microstructure estimate of $K_T$

- Upstream of Drake Passage $K_T^{tracer}$ and $K_T^\epsilon$ are similar
- Downstream of Drake Passage $K_T^\epsilon$ at mean depth of tracer is an order of magnitude smaller than $K_T^{tracer}$

Mashayek, Ferrari et al., 2017
Modeling approach

- MIT general circulation model of a 1400x400 km patch of Scotia Sea
- Resolution of 500-600 m, 100 vertical levels (30 m resolution at tracer depth)
- Model forced at boundaries with a coarser resolution patch of ACC constrained to observations (Tulloch, Ferrari et al., 2014)
- Smith and Sandwell topography at one minute (1/60th of degree)
- Vertical profile of $K_T$ from microstructure profile imposed everywhere as a function of height above the bottom

Mashayek, Ferrari et al., 2017
Tracer evolution

- Tracer is released along western boundary of domain based on sampling of real tracer along that line
- Movie shows that tracer is
  - stirred laterally by geostrophic eddies toward seamounts/ridges
  - mixed vertically by enhanced $K_T$ next to seamounts/ridges
Numerical estimate of $K_{T}^{tracer}$

- We can estimate the diffusivity experienced by the tracer as

$$\bar{K} \equiv \frac{\iiint K_{T}c \, dV}{\iiint c \, dV}$$

$\bar{K} \approx 30 \times 10^{-5} \text{m}^2\text{s}^{-1}$

$\kappa$ at tracer depth $\approx 2 \times 10^{-5} \text{m}^2\text{s}^{-1}$

*Mashayek, Ferrari et al., 2017*
Numerical estimate of $K_T^{tracer}$

- Panel 1: distribution of tracer peaks ~2300 m above seafloor and decays below
- Panel 2: profile of $K_T$ increases toward seafloor
- Panel 3: increase of $K_T$ is so large that estimate of $\bar{K}$ is dominated by integral over bottom 1000 m
- **Mixing occurs when tracer is within 1000 m of seafloor**

*Mashayek, Ferrari et al., 2017*
Numerical estimate of $K_T^{tracer}$

- Tracer tends to accumulate over remounts and ridges where mixing is strong
  - velocities are weaker close to seafloor
  - bottom enhanced mixing drives tracers toward the seafloor
  - other reasons?

Vertically integrated tracer concentration
Numerical estimate of $K^\text{tracer}_T$

- Simply averaging along density surfaces gives little enhancement (mixing hotspots are too rare)
- Accumulation of tracer near topography is crucial to explain large $K^\text{tracer}_T$ in deep ocean (hotspots trap tracer)
Conclusions

- $K_T^\varepsilon = \Gamma \varepsilon / \partial_z \bar{b}$ increases toward rough seafloor

- $\overline{w'b'} = -\Gamma \varepsilon$ increases in magnitude toward rough seafloor

- mixing is confined within a few hundred meters of seafloor

Waterhouse et al., 2014
Conundrum

- \( \bar{w'b'} = -\Gamma \epsilon \) increases in magnitude toward rough seafloor
- Mixing drives sinking, not upwelling of ocean waters