Melancholia states in the climate system: exploring global instabilities and critical transitions

Valerio Lucarini

University of Reading/University of Hamburg

Thanks:

T. Bodai, R. Boschi, H. Dijkstra, B. Eckhardt, C. Grebogi, A. Gritsun, F. Lunkeit, E. Ott, F. Ragone, A. Tantet, J. Yorke

Les Houches

August 3rd 2017

Universität Hamburg

DER FORSCHUNG | DER LEHRE | DER BILDUNG

University of Reading

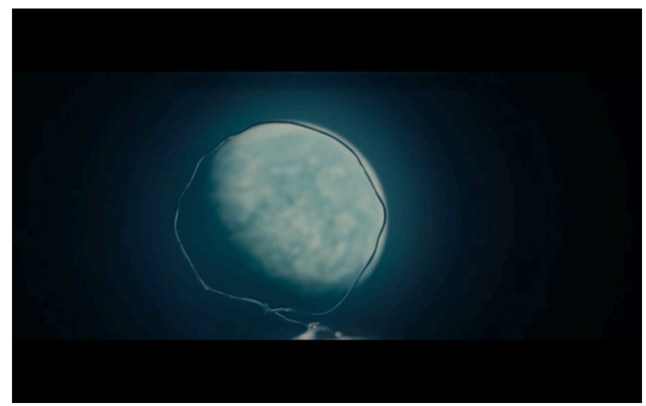
What comes next

- Reading, Aug-Sept 2017
  - CliMathNet Conference (with Ashwin, Bodai, Broecker, Fowler, Freitag, Kuna, Neves, Scott, Shepherd, Williams)
- ICTP, May 2018
  - Advanced Workshop on Nonequilibrium Systems in Physics, Geosciences, and Life Sciences (with Bouchet, Ruffo, Gallavotti, Gambassi)
- Les Houches, Feb-March 2019
  - Physics and Mathematics of Turbulent Flows at Different Scales (with Dubrulle, Faranda, Wouters, Gottwald)
- Inst. Poincare, Autumn 2019
  - The mathematics of climate and the environment (with Ghil, Chekroun, Klein, Le Treut, Speich)

Melancholia (2011, Von Trier)

- Close encounter btw Earth and brown dwarf –Tension grows

- Very close to the tipping point - Tension peaks



- Unavoidable Impact – Tension is released

Motivations: Basics

- Understanding how a system responds to perturbations is a central area of research in natural and mathematical sciences
  - Robustness of the system?
  - Smooth response?
  - Critical Transitions?

- Groundbreaking work by Kubo (1957)
  - Response theory for statistical physical systems
  - Only for near-equilibrium (canonical ensemble)
  - Mathematically and physically non-rigorous, many criticisms
  - Acoustics, Optics, etc. based on Kubo's results
  - Fluctuation-Dissipation Theorem: dictionary between forced and free fluctuations

From Smooth Response …

- Ruelle (‘90s): rigorous response theory for smooth observables of Axiom A systems (eq. & noneq.!)
  - Usual FDT does not apply for nonequilibrium systems: unstable vs stable directions in tangent space
  - Critical Transitions as loss of smoothness in response
  - Theory useful to perform predictions <u>but</u> hard to construct response operator!

- Liverani, Baladi, Dolgopyat,.. (‘00s)
  - Response theory derived using transfer operator approach
  - Change in the invariant measure vs change in observables
  - Beyond Axiom A systems

… to Critical Transitions

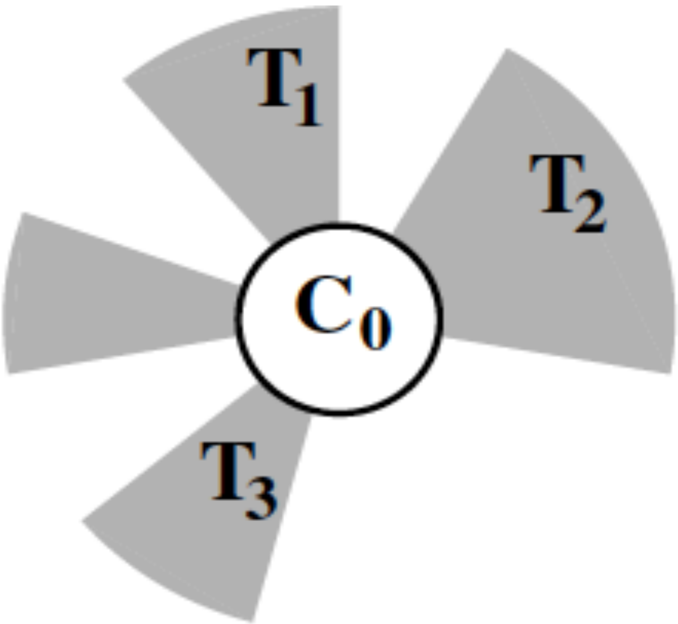
- Catastrophe theory ('60s, Thom, Arnold): comprehensive view of bifurcations in "relatively simple systems'
- Multistability in complex systems & hysteresis
- Defining the boundaries between the basins
- Ashwin et al. 2012: Parameter-, rate-, and noise-induced tipping
- Freidlin-Wentzell theory ('70s) based on Large Deviations Theory ('60s): general laws for noise-induced escape from basins of attraction

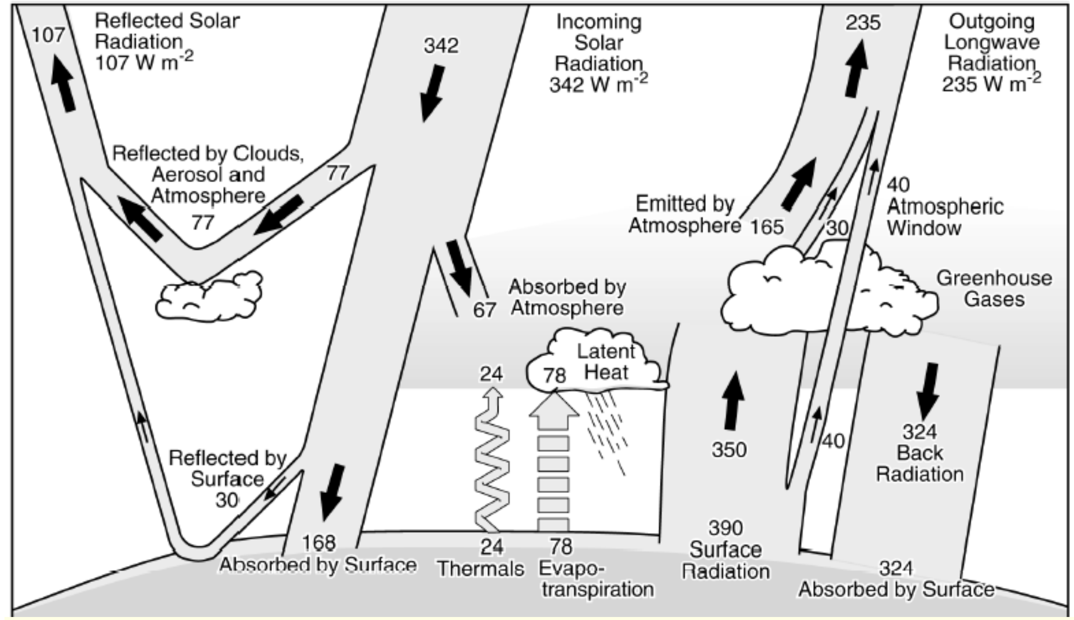
Why Climate is relevant

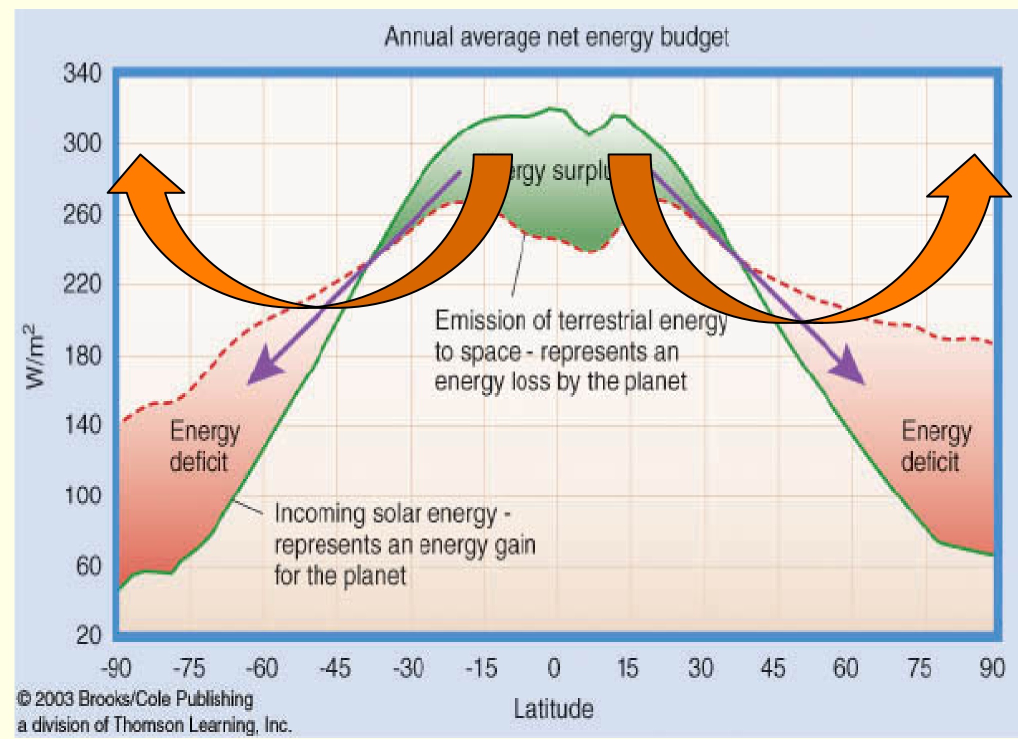
- The climate is a nonequilibrium system whose evolution is driven by inhomogeneous absorption of solar radiation
- The climate features variability of a vast range of spatial and temporal scales
- Understanding the climate response to perturbation is great scientific challenge
  - Anthropogenic climate change
  - Paleoclimate → Life
  - Planetary Science → Habitability
- <u>Smooth vs. nonsmooth response</u>
  - <u>Climate change, climate surprises, climate tipping points</u>

Nonequilibrium

- Vertical





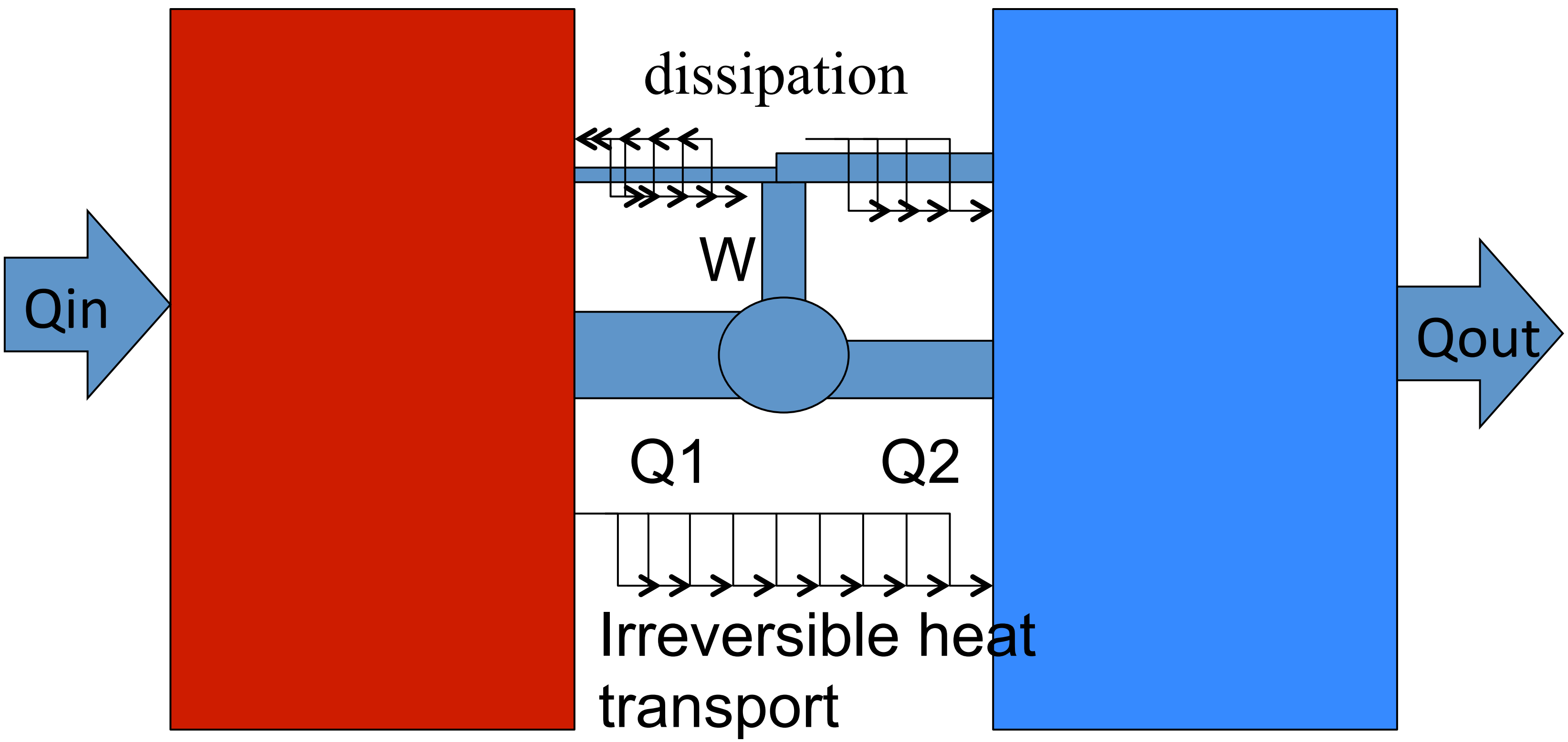


Zonally averaged radiation balance in the atmosphere.

- Horizontal

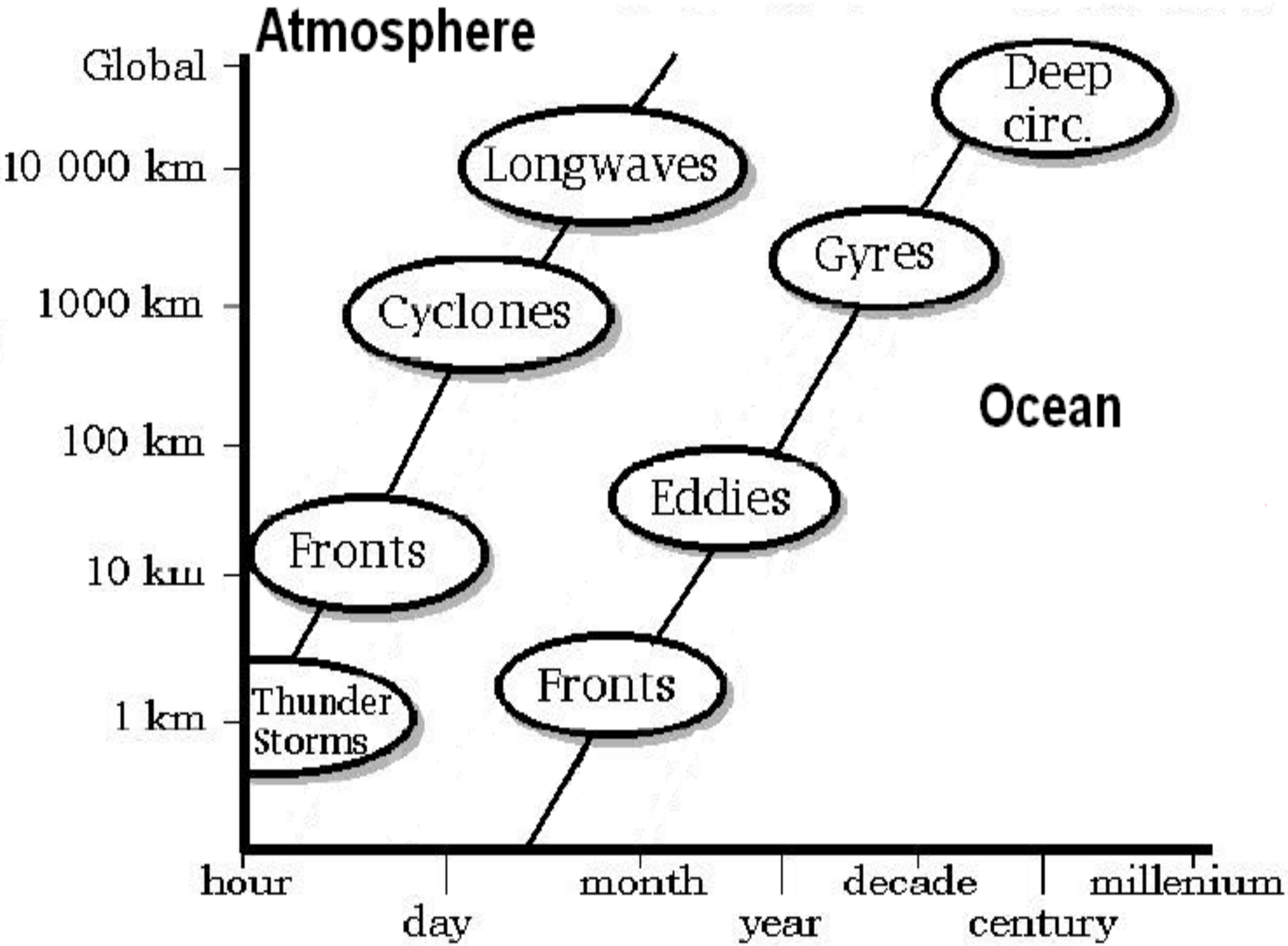
Nonequilibrium

An extremely non-ideal engine



- Efficiency
- Energy transformation
- Entropy Production

Scales of Motions Smagorinsky/Stommel



AGU PUBLICATIONS

Reviews of Geophysics

REVIEW ARTICLE

10.1002/2013RG000446

Mathematical and physical ideas for climate science

Valerio Lucarini[1,2,3], **Richard Blender**[1], **Corentin Herbert**[4], **Francesco Ragone**[1,5], **Salvatore Pascale**[1], **and Jeroen Wouters**[1,6]

[1]Klimacampus, Meteorologisches Institut, University of Hamburg, Hamburg, Germany, [2]Department of Mathematics and Statistics, University of Reading, Reading, UK, [3]Walker Institute for Climate System Research, University of Reading, Reading, UK, [4]National Center for Atmospheric Research, Boulder, Colorado, USA, [5]Klimacampus, Institut für Meereskunde, University of Hamburg, Hamburg, Germany, [6]Laboratoire de Physique, École Normale Supérieure de Lyon, Lyon, France

Key Points:
- Novel selection of mathematical and physical results relevant for GFD
- Frontier theoretical results for testing and improving GFD models
- New results on the use of response theory for climate change

Correspondence to:
V. Lucarini,
valerio.lucarini@uni-hamburg.de

Citation:
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Abstract The climate is a forced and dissipative nonlinear system featuring nontrivial dynamics on a vast range of spatial and temporal scales. The understanding of the climate's structural and multiscale properties is crucial for the provision of a unifying picture of its dynamics and for the implementation of accurate and efficient numerical models. We present some recent developments at the intersection between climate science, mathematics, and physics, which may prove fruitful in the direction of constructing a more comprehensive account of climate dynamics. We describe the Nambu formulation of fluid dynamics and the potential of such a theory for constructing sophisticated numerical models of geophysical fluids. Then, we focus on the statistical mechanics of quasi-equilibrium flows in a rotating environment, which seems crucial for constructing a robust theory of geophysical turbulence. We then discuss ideas and methods suited for approaching directly the nonequilibrium nature of the climate system. First, we describe some recent findings on the thermodynamics of climate, characterize its energy and entropy budgets, and discuss related methods for intercomparing climate models and for studying tipping points. These ideas can also create a common ground between geophysics and astrophysics by suggesting general tools for studying exoplanetary atmospheres. We conclude by focusing on nonequilibrium statistical mechanics, which allows for a unified framing of problems as different as the climate response to forcings, the effect of altering the boundary conditions or the coupling between geophysical flows, and the derivation of parametrizations for numerical models.

Climate Response

A. Smooth response – Response Theory

Constructing the sensitivity of the climate and the measure of the pullback attractor. Link between climate variability and climate change?

B. High sensitivity – Ruelle-Pollicott Resonances

Rough dependence on system's parameters

C. Critical Transitions

Crisis of the high-dimensional attractor

A

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Predicting Climate Change Using Response Theory: Global Averages and Spatial Patterns

Valerio Lucarini[1,2] · Francesco Ragone[3] · Frank Lunkeit[1]

Received: 22 December 2015 / Accepted: 17 March 2016

Abstract The provision of accurate methods for predicting the climate response to anthropogenic and natural forcings is a key contemporary scientific challenge. Using a simplified and efficient open-source general circulation model of the atmosphere featuring $O(10^5)$ degrees of freedom, we show how it is possible to approach such a problem using nonequilibrium statistical mechanics. Response theory allows one to practically compute the time-dependent measure supported on the pullback attractor of the climate system, whose dynamics is non-autonomous as a result of time-dependent forcings. We propose a simple yet efficient method

Constructing the time-dependent measure via Ruelle Response Theory

- Perturbation to Axiom A: $\dot{x} = \tilde{F}(x,t) = F(x) + \varepsilon e(t) X(x)$
- Change in expectation value of a smooth Φ:

$$\langle \Phi \rangle (t) = \langle \Phi \rangle_0 + \sum_{k=1}^{\infty} \varepsilon^k \langle \Phi \rangle^{(k)} (t)$$

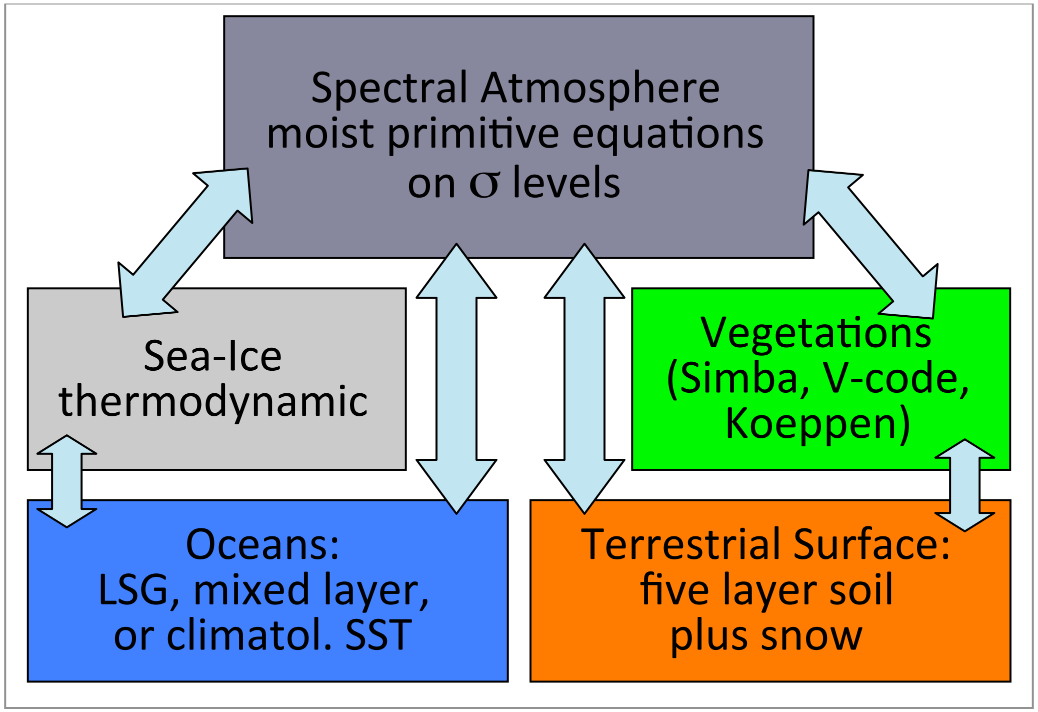
Pullback attractor

- Linear term: $\langle \Phi \rangle^{(1)} (t) = \int d\sigma G_\Phi^{(1)} (\sigma) e(t-\sigma)$
- Linear Green: $G_\Phi^{(1)} (t) = \int \rho_0 (dx) \Theta(t) X \cdot \nabla S^t \Phi$
- Linear suscept: $\chi_\Phi^{(1)} (\omega) = \int dt \exp[i\omega t] G_\Phi^{(1)} (t)$

  obeys KK relations

$$\langle \Phi \rangle^{(1)} (\omega) = \chi_\Phi^{(1)} (\omega) e(\omega)$$

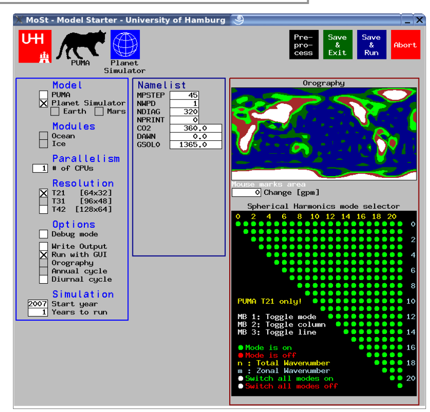
PLASIM: An efficient Climate Model

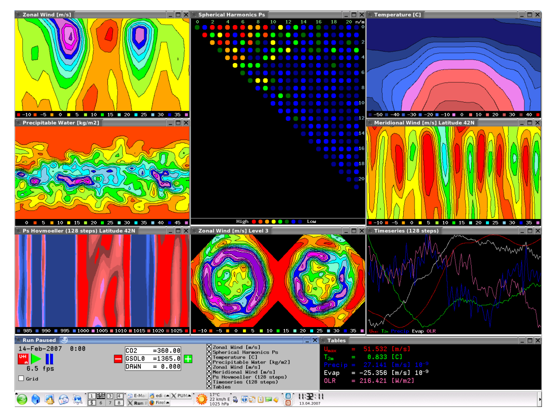


Key features

- portable
- fast
- open source
- parallel
- modular
- easy to use
- documented
- compatible

Model Starter
and
Graphic User Interface





$O(10^5)$ d.o.f.

Universität Hamburg

KlimaCampus

Step 1

- Observable: globally averaged T_S
- Forcing: increase of CO_2 concentration $\quad f(t)$
- Linear response: $\langle T_S \rangle_f^{(1)}(t) = \int d\sigma G_{T_S}^{(1)}(\sigma) f(t-\sigma)$
- We perform ensemble experiments
  - Concentration $\uparrow$ at t=0 $\quad f(t) = \varepsilon\Theta(t)$
- Fantastic, we estimate

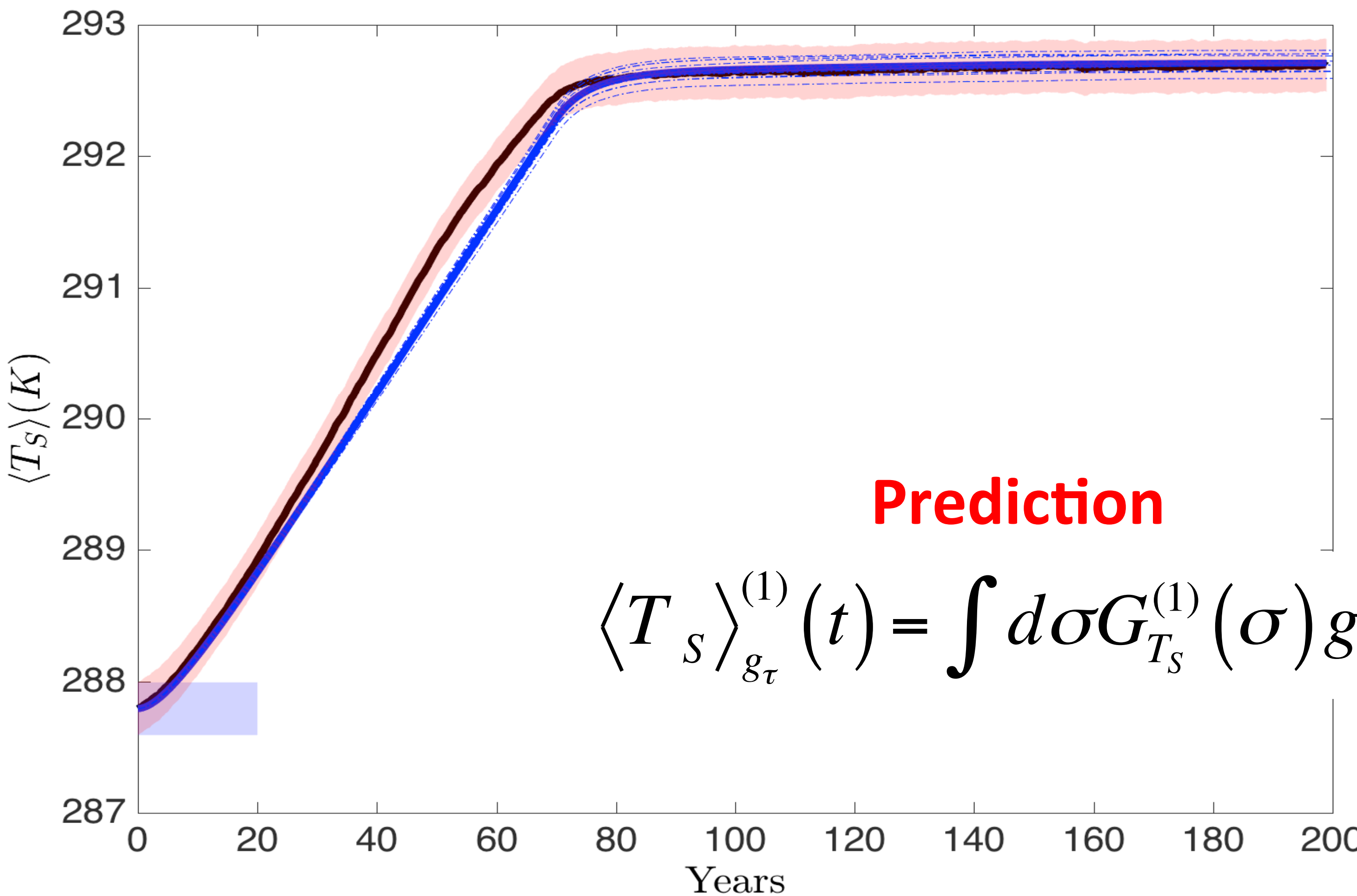
$$\frac{d}{dt}\langle T_S \rangle_f^{(1)}(t) = \varepsilon G_{T_S}^{(1)}(t)$$

- …and we predict:

$$\langle T_S \rangle_g^{(1)}(t) = \int d\sigma G_{T_S}^{(1)}(\sigma) g(t-\sigma)$$

Climate Change Prediction - T_S

$[CO_2]$ 360 ppm → 720 ppm at 1% per year
2X after $\tau \approx$ 70 years, constant after that



Prediction

$$\left\langle T_S \right\rangle_{g_\tau}^{(1)}(t) = \int d\sigma G_{T_S}^{(1)}(\sigma) g_\tau(t-\sigma)$$

(Transient) Climate Sensitivity

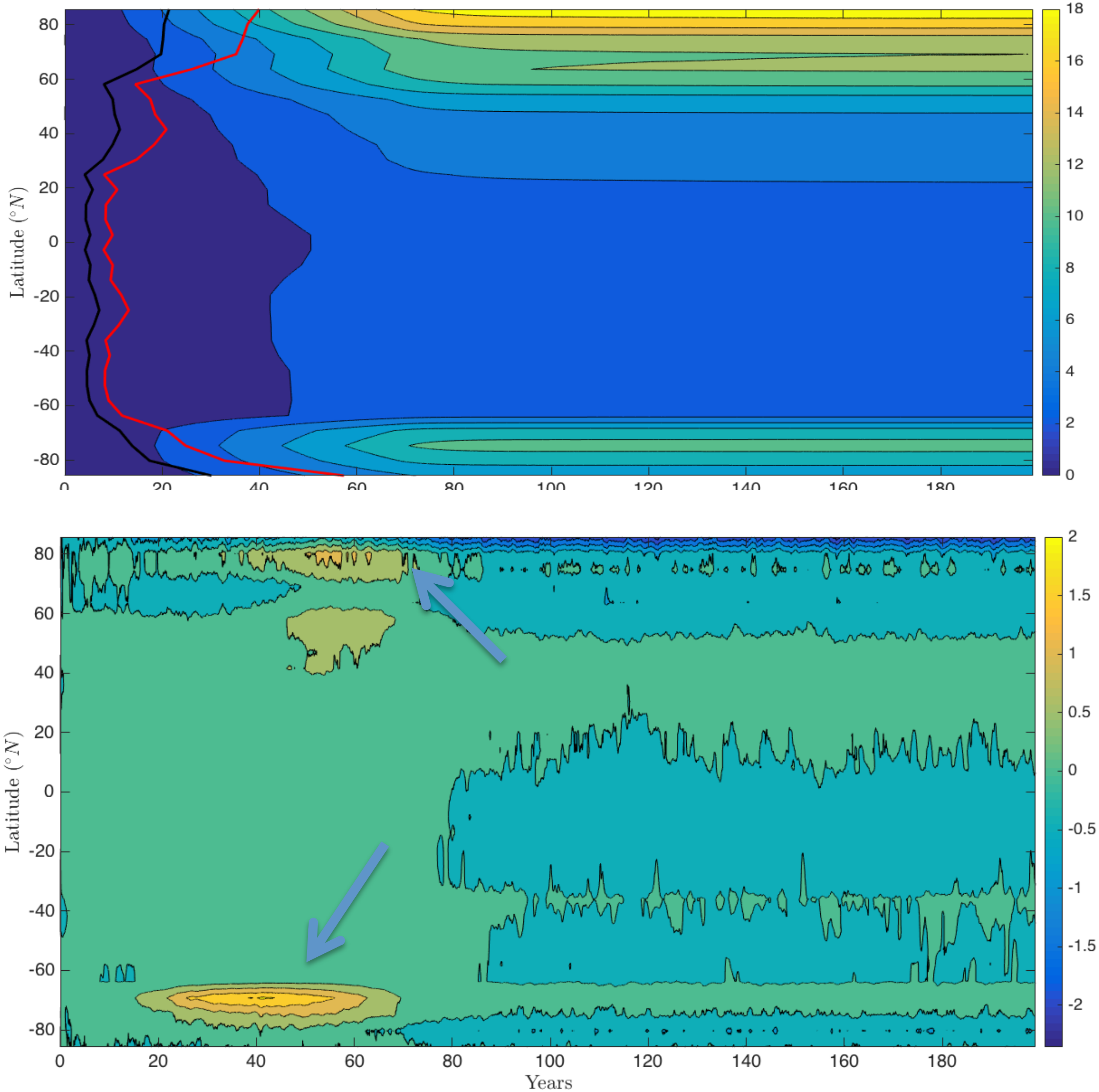
$$ECS = \Re\left\{\chi_{T_S}^{(1)}(0)\right\} = \frac{2}{\pi}\int d\omega \,\mathrm{Re}[\langle T_s\rangle^{(1)}(\omega)]$$ “EQUILIBRIUM”

$$TCR(\tau) = \langle T_S\rangle_{g_\tau}^{(1)}(\tau) = \int d\sigma G_{T_S}^{(1)}(\sigma)\, g_\tau(\tau-\sigma)$$ “TRANSIENT”

$$= ECS - P\int_{-\infty}^{+\infty} f_{CO_2}^{2x}\chi_{T_s}^{(1)}(\omega)\frac{1+\mathrm{sinc}(\omega\tau/2)\mathrm{e}^{-i\omega\tau/2}}{2\pi i\omega}d\omega$$

- ΔT at the end of the ramp (τ=70 ys)
- Smaller than ECS (4.1 K vs 4.8 K)
- Want to define inertia at all values of τ
  - Instantaneous vs quasi static

Beyond Global Indicators



T_S Projection

Error

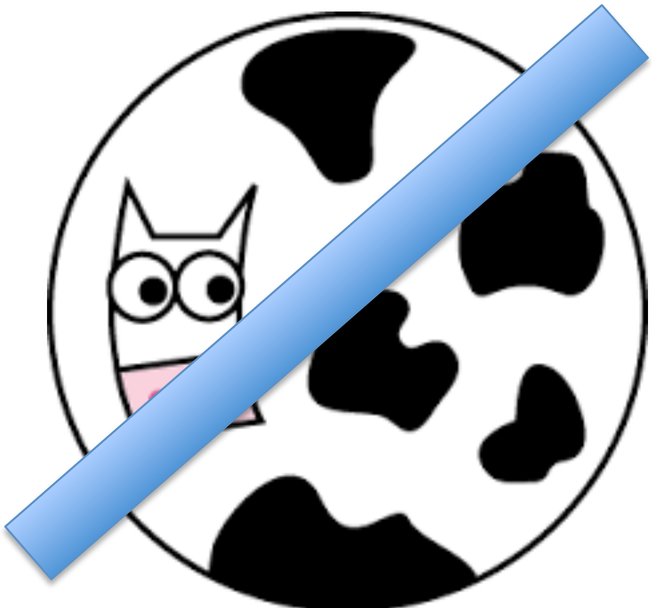
Nonlinear Process

Ice-Albedo

Common Sense

- Forced fluctuations will project on the free ones
  - FDT will work
- … unless you are in low dimension and/or use cooked-up observables & forcings
- Past experiments: sometimes FDT works, sometimes it does not work. Why so?

- .. But sometimes cows are not really spherical ...



Gritsun & Lucarini, 2017, Physica D

A

Physica D 349 (2017) 62–76

Contents lists available at ScienceDirect

Physica D

journal homepage: www.elsevier.com/locate/physd

Fluctuations, response, and resonances in a simple atmospheric model

Physica D 349, 62-76 (2017)

Andrey Gritsun[a,b,*], Valerio Lucarini[c,d,e]

[a] Institute of Numerical Mathematics, Russian Academy of Sciences, Moscow, Russian Federation
[b] Institute of Applied Physics, Russian Academy of Sciences, Nizhny Novgorod, Russian Federation
[c] Department of Mathematics and Statistics, University of Reading, United Kingdom
[d] Centre for the Mathematics of the Planet Earth, University of Reading, United Kingdom
[e] CEN, University of Hamburg, Hamburg, Germany

HIGHLIGHTS

- We study the response of an atmospheric model to perturbations of its parameters.
- Methods of nonequilibrium statistical mechanics, Ruelle response theory are used.
- Response could be very different from the natural variability, the FDT is violated.
- Unexpected behavior for forcings with strong projections on the stable manifold.
- Resonant system behavior could be explained in terms of unstable periodic orbits.

ARTICLE INFO

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Communicated by H.A. Dijkstra

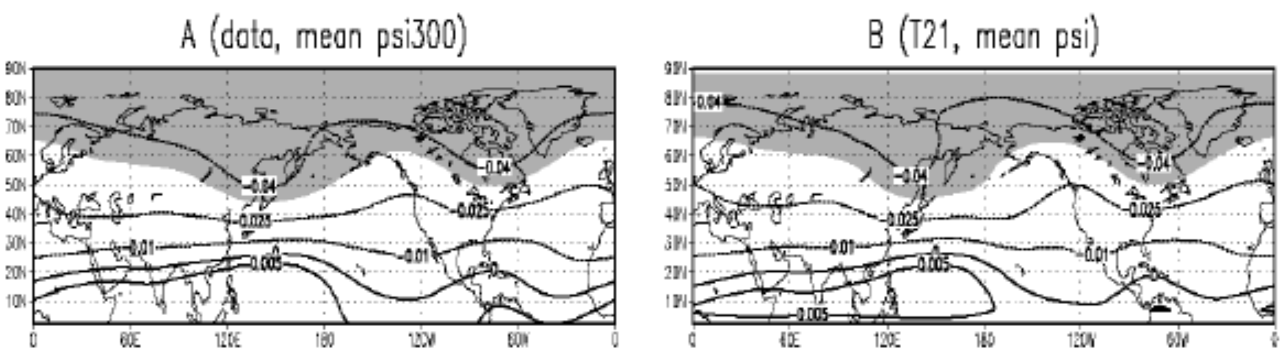
ABSTRACT

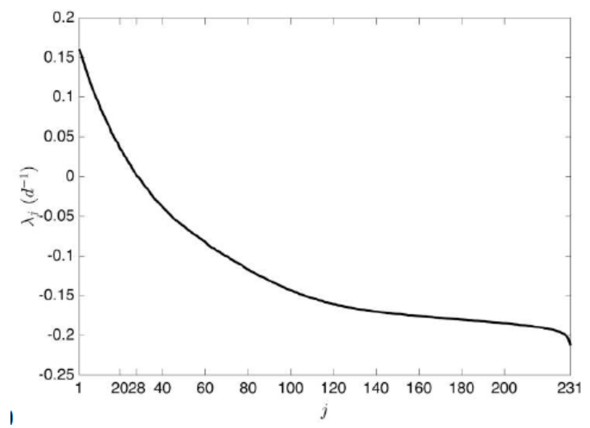
We study the response of a simple quasi-geostrophic barotropic model of the atmosphere to various classes of perturbations affecting its forcing and its dissipation using the formalism of the Ruelle response theory. We investigate the geometry of such perturbations by constructing the covariant Lyapunov vectors of the unperturbed system and discover in one specific case – orographic forcing – a substantial projection of the forcing onto the stable directions of the flow. This results into a resonant response shaped as a Rossby-like wave that has no resemblance to the unforced variability in the same range of spatial and temporal scales. Such a climatic surprise corresponds to a violation of the fluctuation–dissipation

Simple model of the mid-latitudes

$$\frac{\partial \Delta\Psi}{\partial T} + J(\Psi, \Delta\Psi + L + K_H H) = -A\Delta\Psi + M\Delta^2\Psi + F_{ext}$$

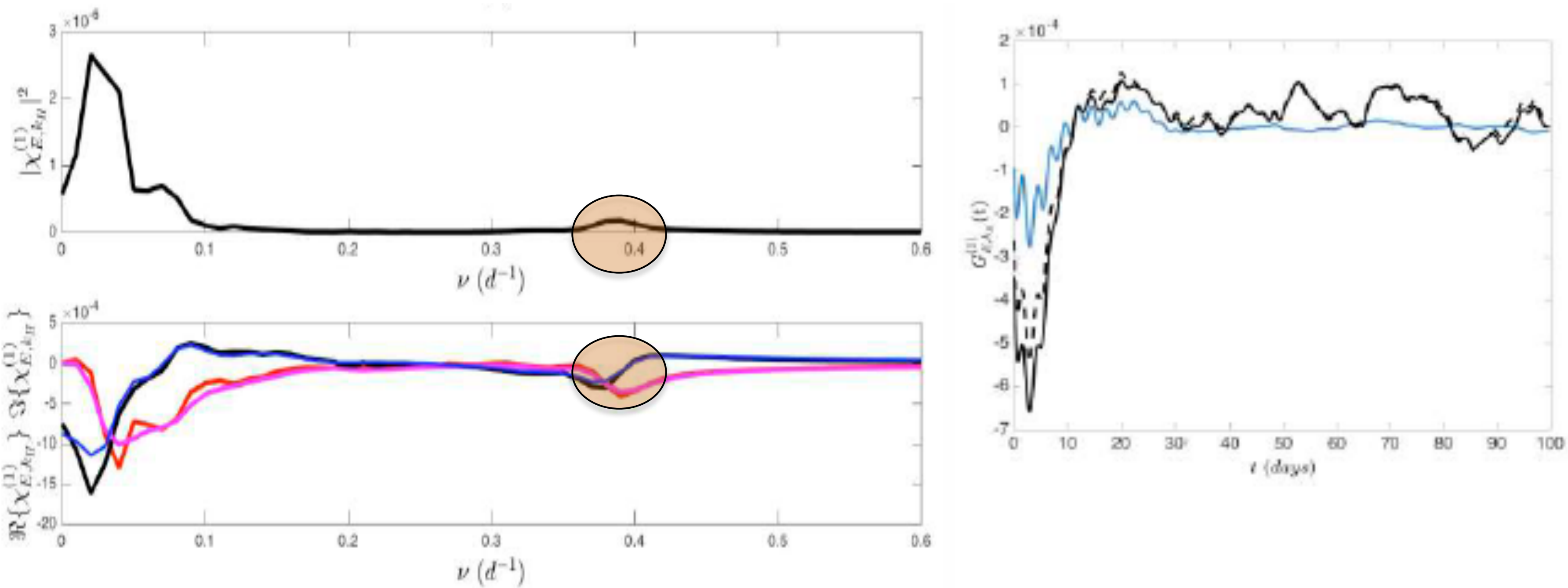
- Ψ is streamfunction, $\Delta\Psi$ vorticity
- Rotation, Orographic forcing, diffusion, friction
- External driving
- Made to look like winter atmosphere
- Very chaotic (#(λ_j,>0)=28, 231 dof)





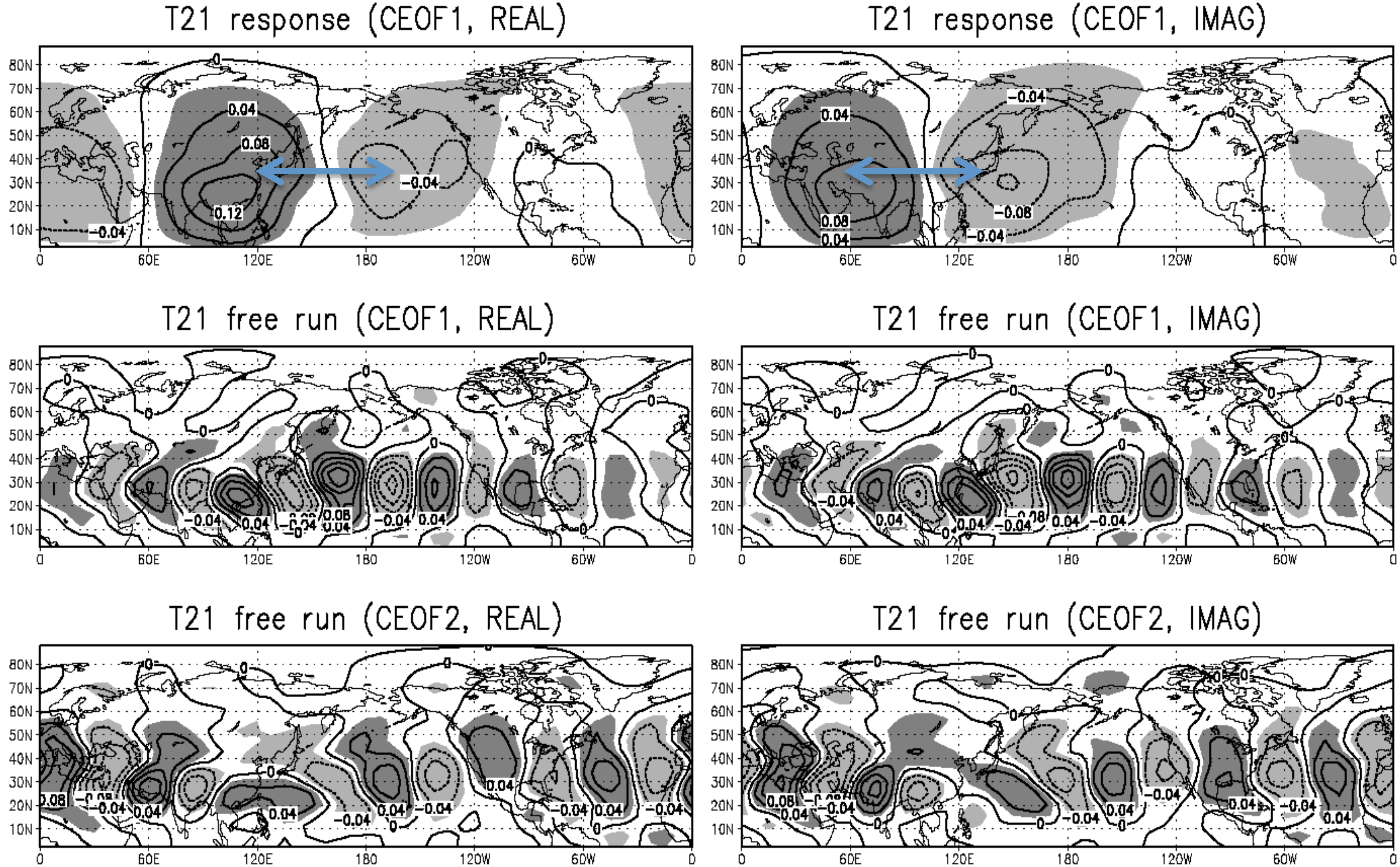
Response to Forcings

- Response orographic forcing vs natural variability
- Resonance inexplicable with FDT

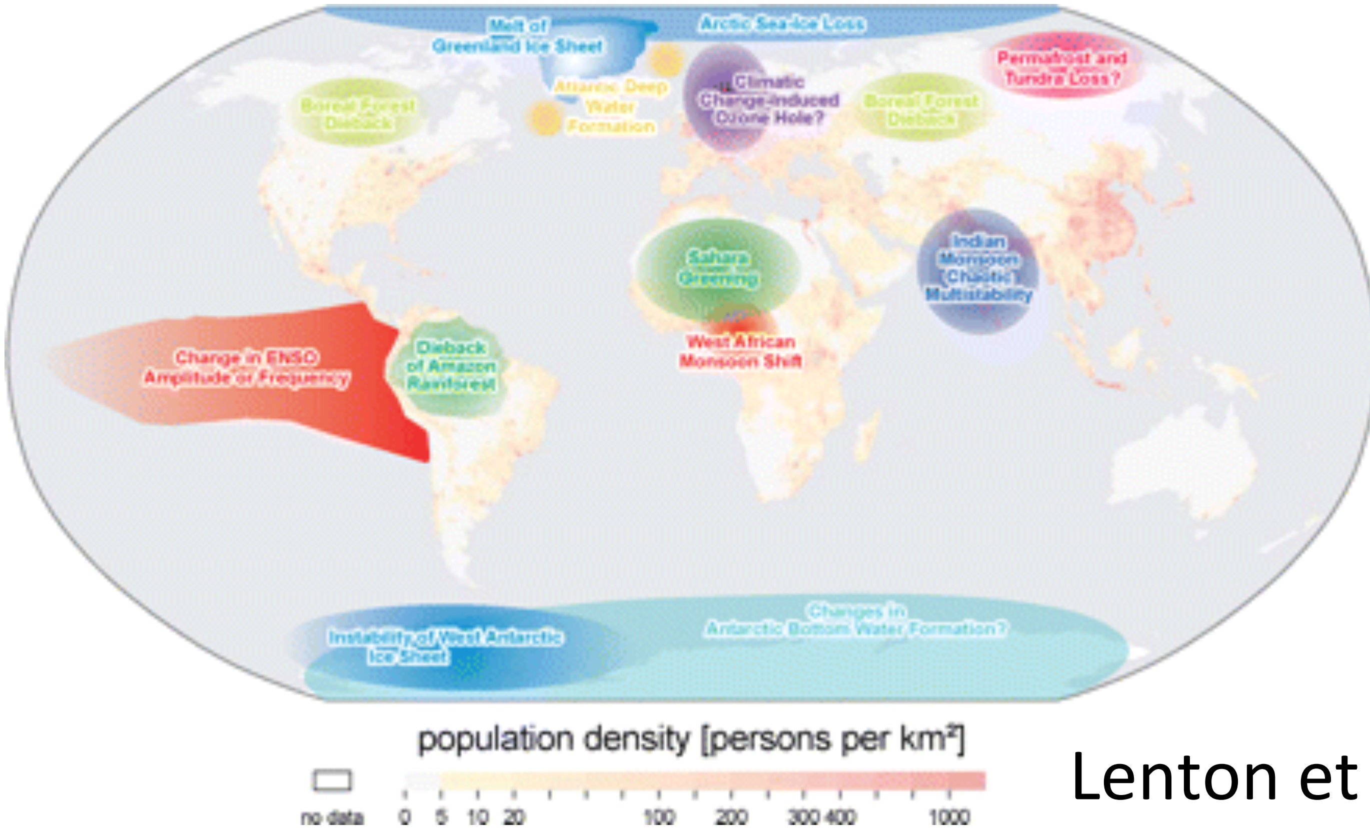


- Resonance comes from a group of UPOs
- UPOs rarely visited by system but resonant
- Possible paradigm for **climatic surprises**

Effect of Orographic forcing



"Tipping elements"

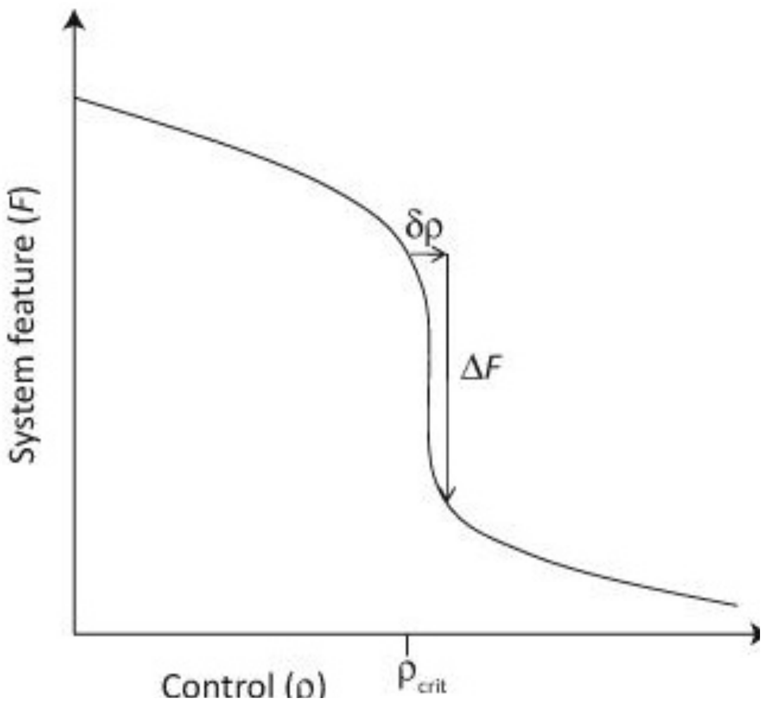


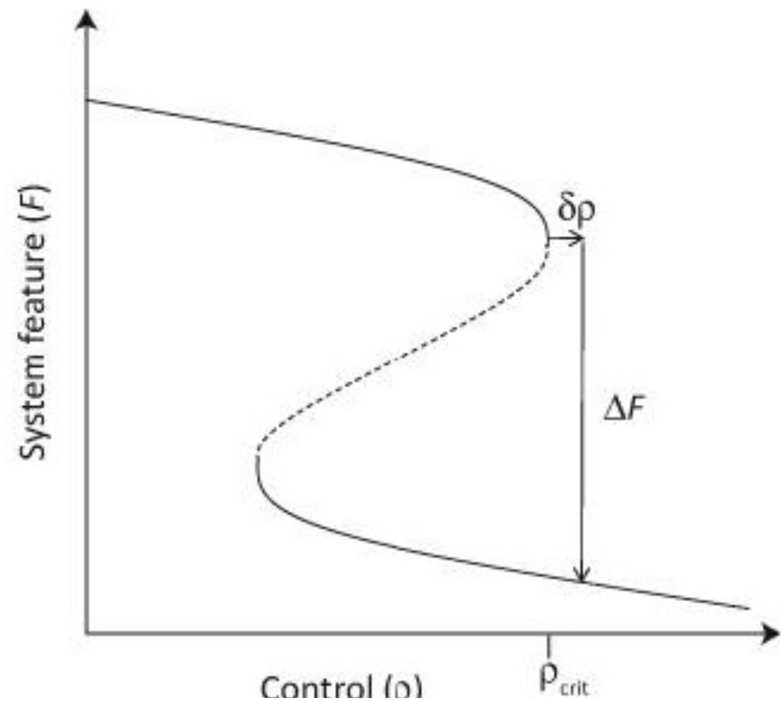
Lenton et al. 2007

- "Highly sensitive" regions to climate change, "irreversible" response to forcings


WARNING!!
CLIMATE
TIPPING
POINT AHEAD

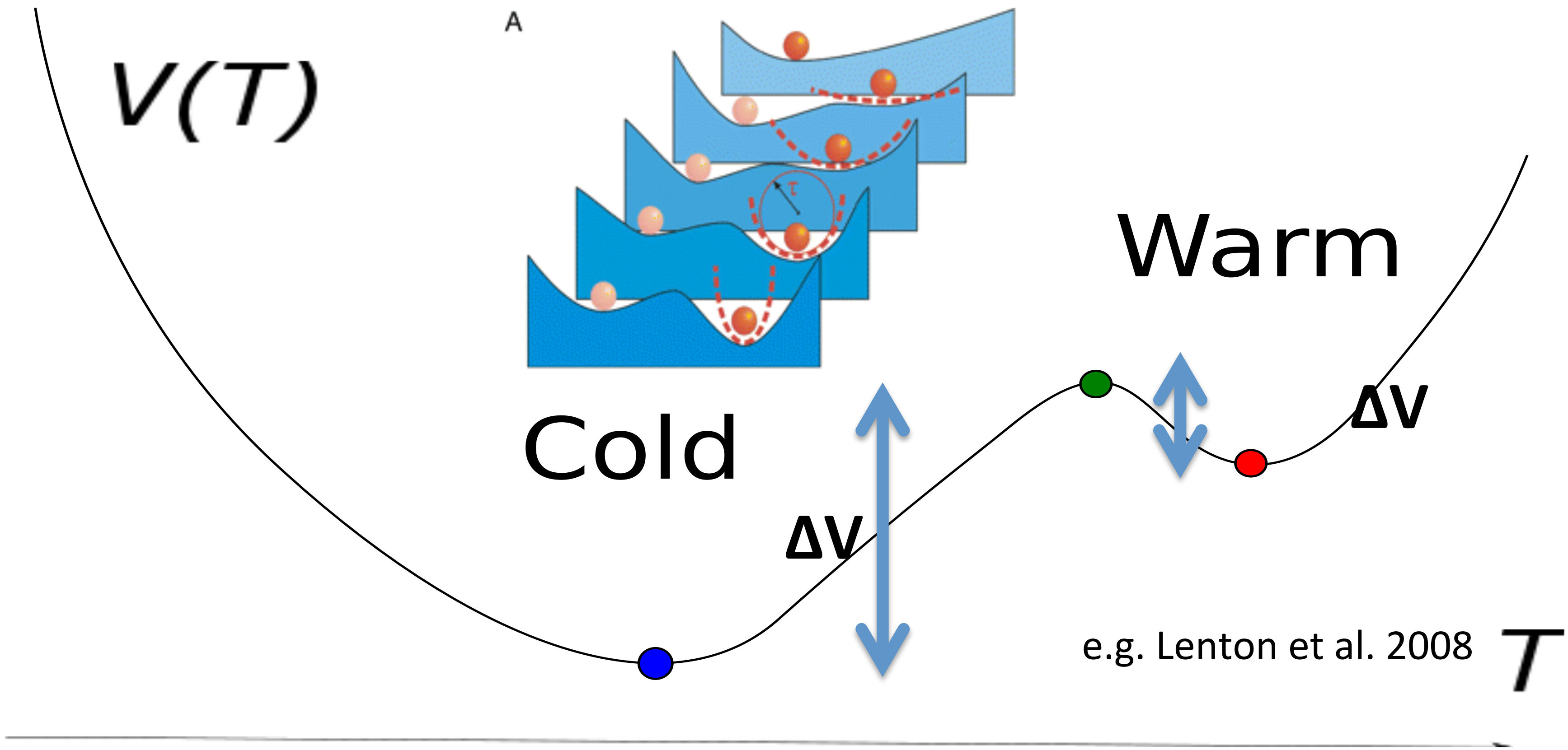
High Sensitivity vs Tipping Point





Stolen from Lenton, somewhere…

Effective Potential plus noise



- A simple model of the form $\mathrm{d}Y = -\mathrm{d}V/\mathrm{d}Y + \alpha \mathrm{d}W$
- Transitions: noise acting on the effective potential
- Prob ≈ $\exp[-2\Delta V/\varepsilon^2]$
- Time series analysis vs dynamics: Procedure is not robust
  - Unless: EVT - Faranda, Lucarini, Manneville, Wouters 2014

Climate Response

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Rough dependence on system's parameters

C. Critical Transitions

Crisis of the high-dimensional attractor

B

Crisis of the Chaotic Attractor of a Climate Model: A Transfer Operator Approach

Alexis Tantet

Institute of Marine and Atmospheric research Utrecht, Department of Physics and Astronomy, University of Utrecht, Utrecht, The Netherlands

E-mail: a.j.j.tantet@uu.nl

Valerio Lucarini[1,2]

[1]Meteorological Institute and Centre for Earth System Research and Sustainability (CEN), University of Hamburg, Hamburg, Germany
[2]Department of Mathematics and Statistics, University of Reading, Reading, UK

Frank Lunkeit

Meteorological Institute and Centre for Earth System Research and Sustainability (CEN), University of Hamburg, Hamburg, Germany

Henk A. Dijkstra

Institute of Marine and Atmospheric research Utrecht, Department of Physics and Astronomy, University of Utrecht, Utrecht, The Netherlands

Abstract.

The destruction of a chaotic attractor leading to a rough change in the dynamics of a system as a control parameter is smoothly varied is studied. While bifurcations involving non-chaotic invariant sets, such as fixed points or periodic orbits, can be characterised by a Lyapunov exponent crossing the imaginary axis, little is known about the changes in the Lyapunov spectrum of chaotic attractors during a crisis, notably because chaotic invariant sets have positive Lyapunov exponents and because the Lyapunov spectrum varies on the invariant set. However, one would expect the critical slowing down of trajectories observed at the approach of a classical bifurcation to persist in the case of a chaotic attractor crisis. The reason is that, as the system becomes susceptible to the physical instability mechanism responsible for the crisis, it turns out to be less and less resilient to exogenous perturbations and to spontaneous fluctuations due to other types of instabilities on the attractor.

The statistical physics framework, extended to nonequilibrium systems, is particularly well suited for the study of global properties of chaotic systems. In particular, the semigroup of transfer operators governing the finite time evolution of probability distributions in phase space and its spectrum characterises both the relaxation rate of distributions to a statistical steady-state and the stability of this steady-state to perturbations. If critical slowing down indeed occurs in the approach to an attractor crisis, the gap in the spectrum (between the leading eigenvalue and the secondary ones) of the semigroup of transfer operators is expected to shrink.

B

Resonances in a Chaotic Attractor Crisis of the Lorenz Flow

Alexis Tantet · Valerio Lucarini · Henk A. Dijkstra

May 24, 2017

Abstract Local bifurcations of stationary points and limit cycles have successfully been characterized in terms of the critical exponents of these solutions. Lyapunov exponents and their associated covariant Lyapunov vectors have been proposed as tools for supporting the understanding of critical transitions in chaotic dynamical systems. However, it is in general not clear how the statistical properties of dynamical systems change across a boundary crisis during which a chaotic attractor collides with a saddle.

This behavior is investigated here for a boundary crisis in the Lorenz flow, for which neither the Lyapunov exponents nor the covariant Lyapunov vectors provide a criterion for the crisis. Instead, the convergence of the time evolution of probability densities to the invariant measure, governed by the semigroup of transfer operators, is expected to slow down at the approach of the crisis. Such convergence is described by the eigenvalues of the generator of this semigroup, which can be divided into two families, referred to as the *stable* and *unstable* Ruelle-Pollicott resonances, respectively. The former describes the convergence of densities to the attractor (or escape from a repeller) and is estimated from many short time series sampling the phase space. The latter is responsible for the decay of correlations, or mixing, and can be estimated from a long times series, invoking ergodicity.

It is found numerically for the Lorenz flow that the stable resonances do approach the imaginary axis during the crisis, as is indicative of the loss of global

A. Tantet · V. Lucarini
Universität Hamburg, Center for Earth System Research and Sustainability, Meteorologisches Institut, Hamburg, Germany
E-mail: alexis.tantet@uni-hamburg.de

V. Lucarini
Department of Mathematics and Statistics, University of Reading, Reading, UK

V. Lucarini
Centre for the Mathematics of Planet Earth, University of Reading, Reading, UK

H. A. Dijkstra
Institute for Marine and Atmospheric research Utrecht, Department of Physics and Astronomy, University of Utrecht, Utrecht, The Netherlands

What happens

Near Critical Transition two separate processes:

- Critical Slowing down: the decay of correlations becomes slower and slower
  - Property of the attractor
  - The system has longer memory
  - The response to perturbation diverges
  - Radius of expansion of Ruelle's theory
- Convergence of ensembles: the attractor attracts less efficiently nearby trajectories
  - Property of neighborhood of the attractor
  - Cannot be flagged by dynamics on the attractor

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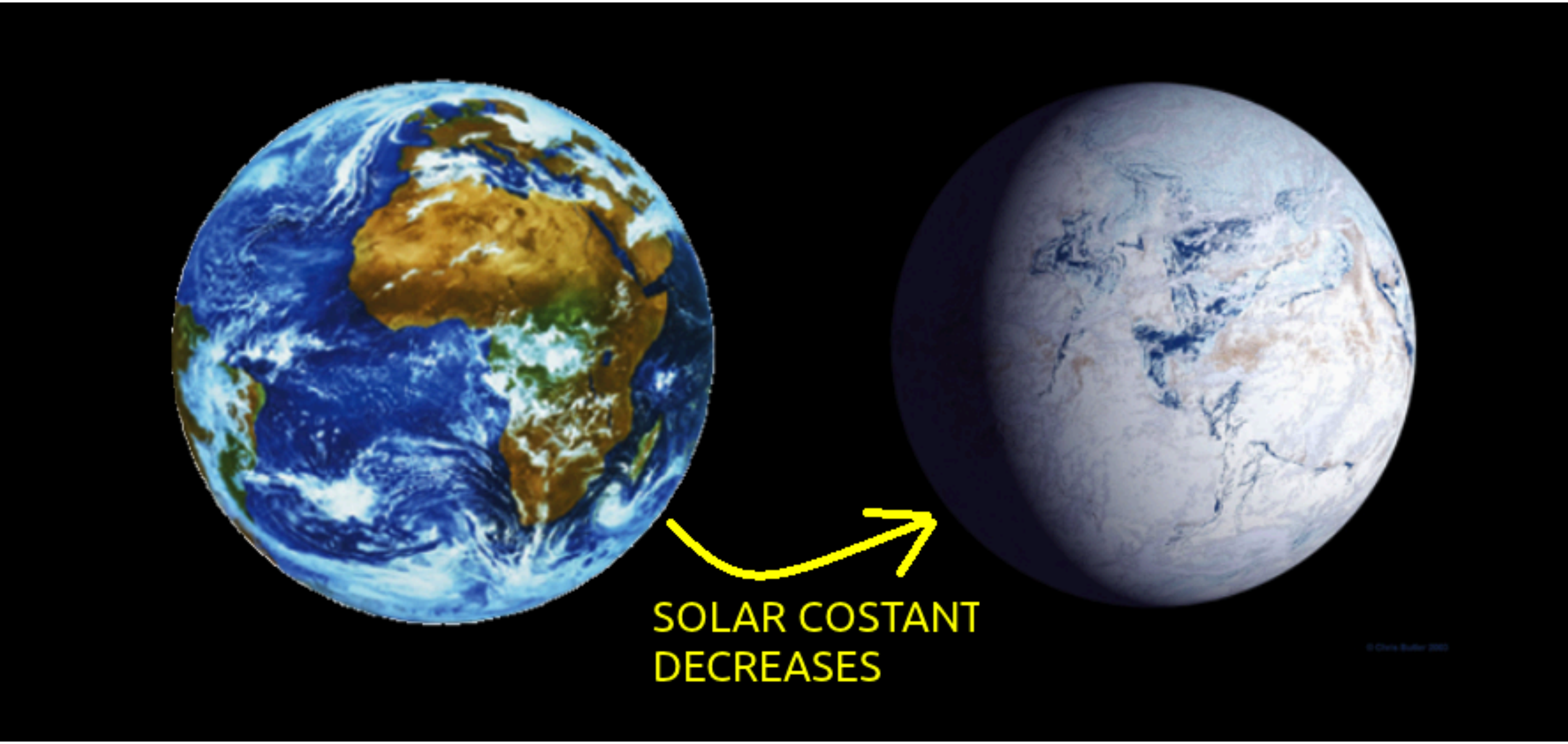
Snowball/Snowfree Transitions



- … and the reserved transition

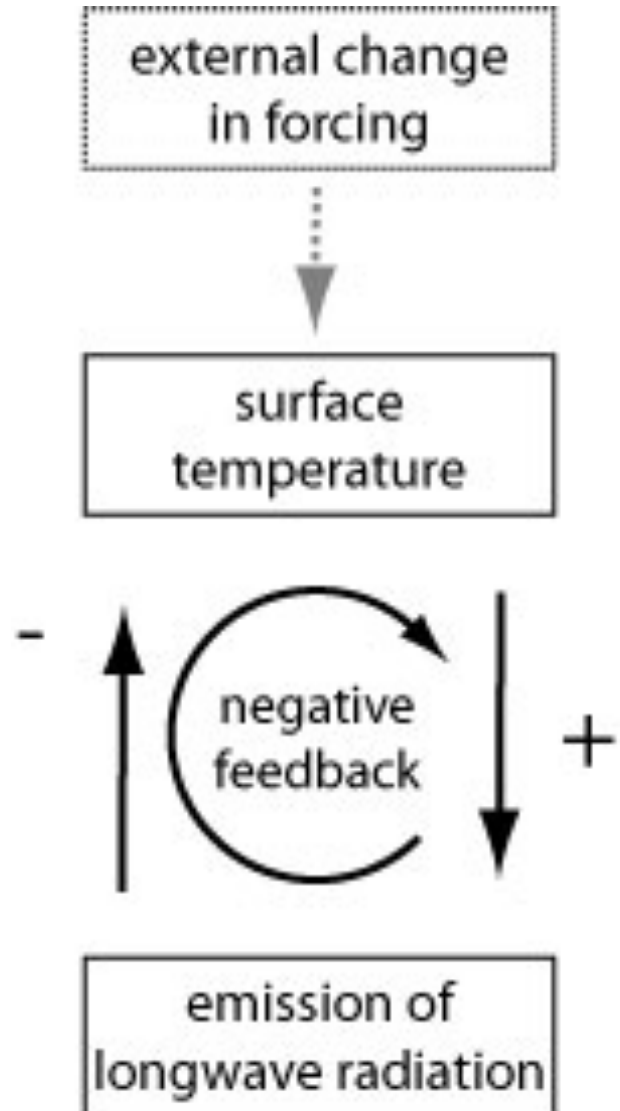
Historical Note

- The bistability of the Earth system was discovered when studying the possible effects of the nuclear winter
- Budyko, Sellers in the late ‘60 realized that a prolonged nuclear winter might lead to a global glaciation
- Ghil (1976) extended the analysis
- People laughed at this possibility… but in early ’90s paleo evidences emerged!
  - Beware critical transitions!

Feedbacks (1)

- Radiative feedback
- Warmer bodies emit more
- Cooler bodies emit less

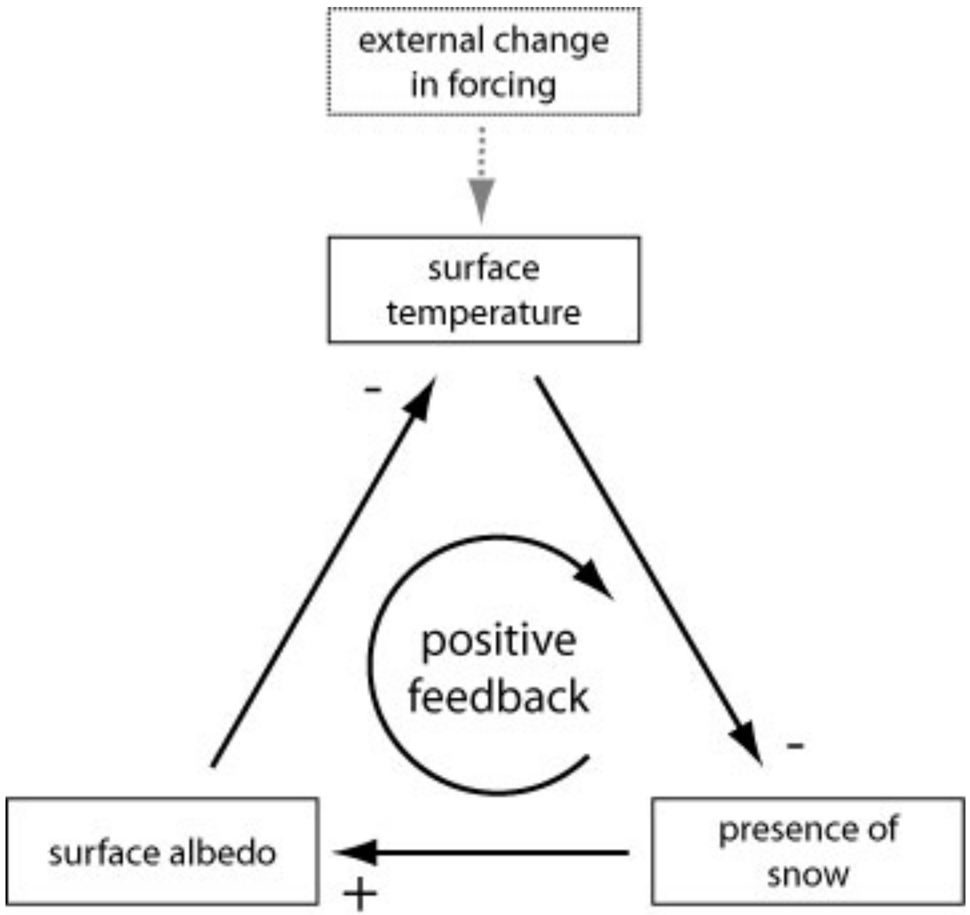
- Negative feedback



Feedbacks (2)

- Ice-albedo feedback:
- A warmer surface has less snow cover
- Albedo decreases
- More radiation is absorbed
- Temperature anomaly is strengthened

- Positive feedback

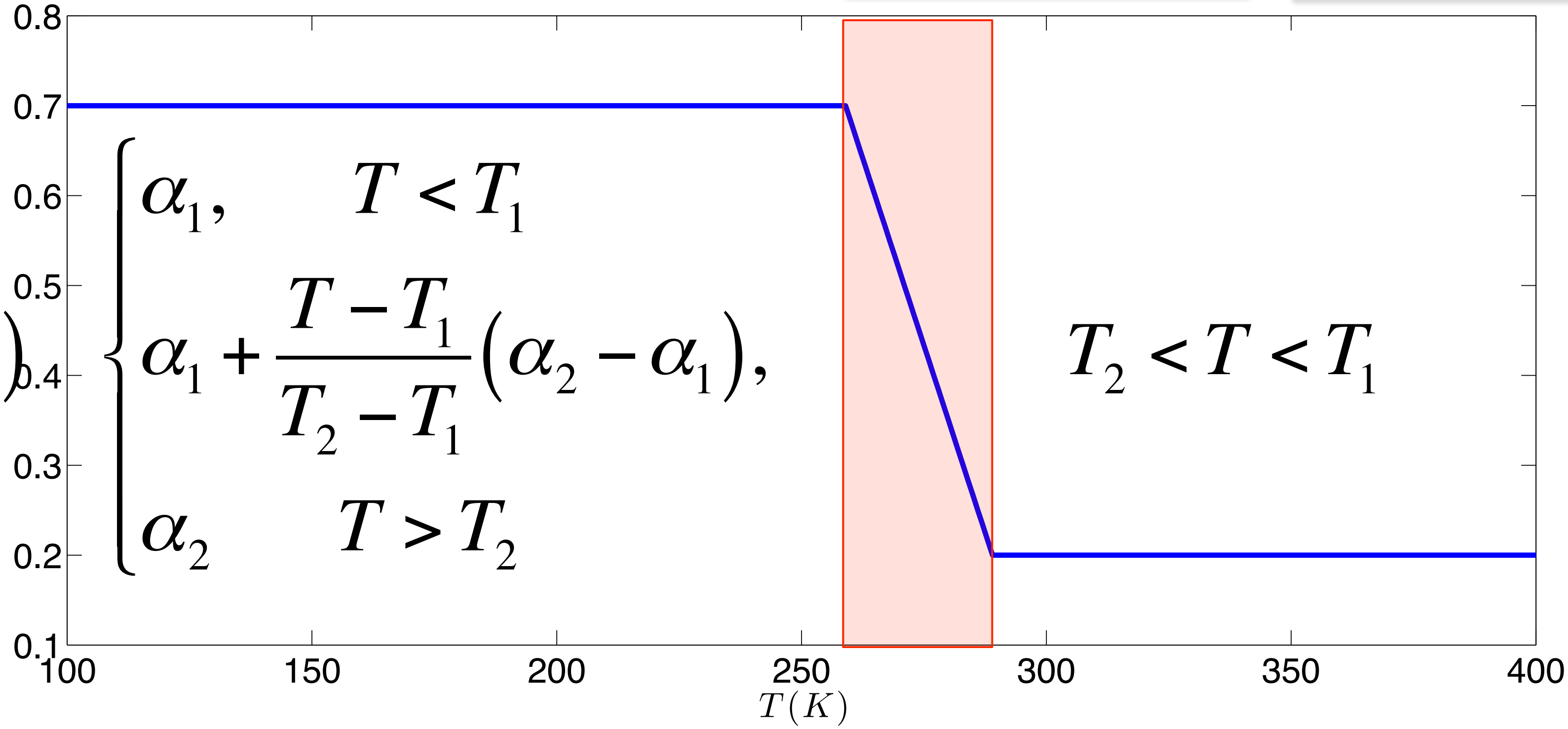


0-D Energy balance

Ice/Albedo Feedback

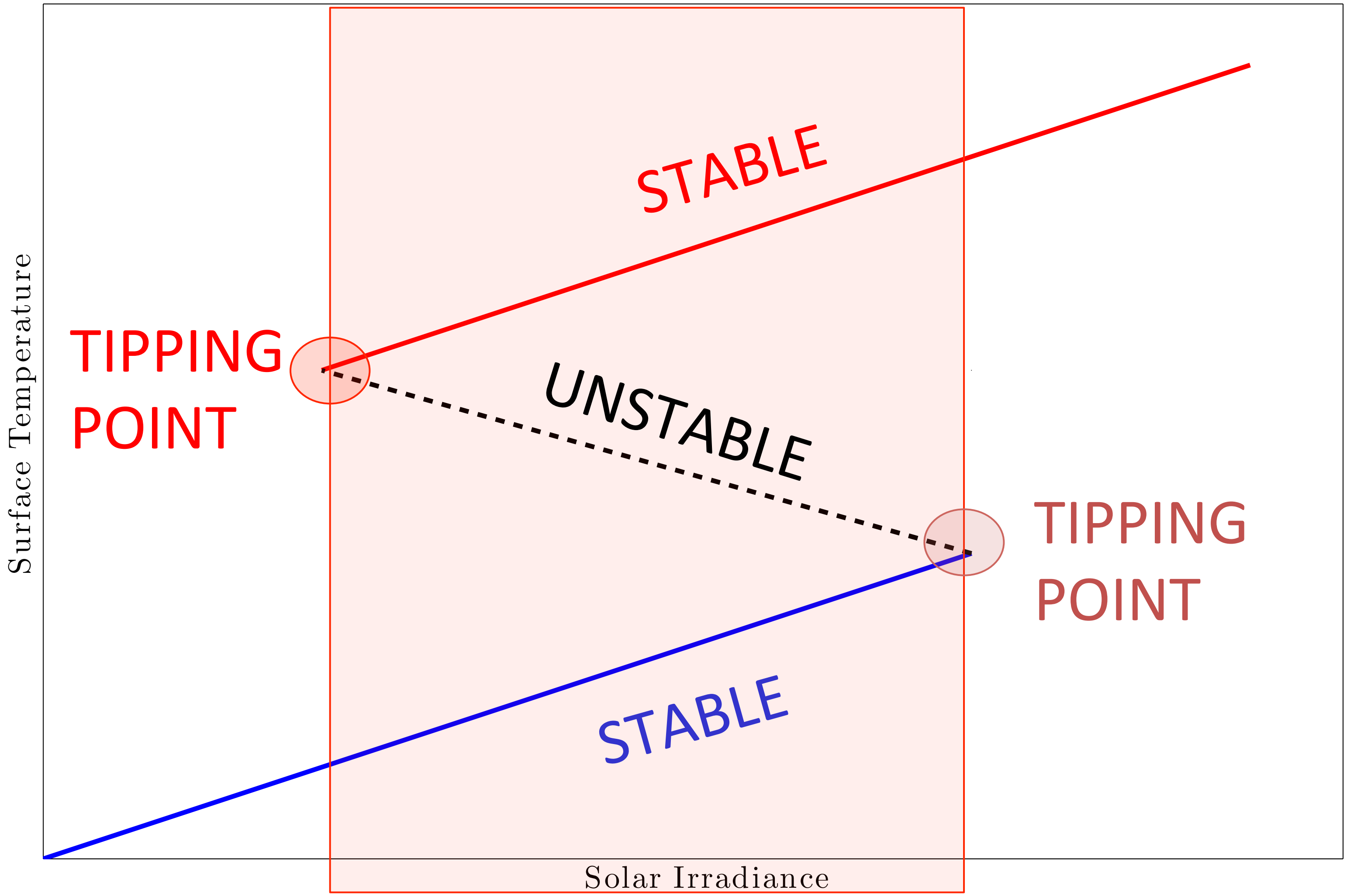
- Energy balance : $$C\frac{dT}{dt} = I - O$$

$$C\frac{dT}{dt} = \underbrace{\left(1-\alpha_p(T)\right)\frac{S}{4}}_{I} - \underbrace{\left(A+BT\right)}_{O}$$



$$\alpha_p(T) \begin{cases} \alpha_1, & T < T_1 \\ \alpha_1 + \dfrac{T-T_1}{T_2-T_1}\left(\alpha_2-\alpha_1\right), & T_2 < T < T_1 \\ \alpha_2 & T > T_2 \end{cases}$$

Bifurcations



Bistability

1D Energy Balance Model – Ghil '76

$$c(x)T_t =$$

$$\left(\frac{2}{\pi}\right)^2 \frac{1}{\cos(\pi x/2)}[\cos(\pi x/2)k(x,T)T_x]_x$$ Diffusion

$$+\mu Q(x)[1-\alpha(x,T)]$$ Incoming Radiation

$$-\sigma T^4[1-m\tanh(c_3 T^6)]$$ Outgoing Radiation

$$T_x(-1,t) = T_x(1,t) = 0, \quad T(x,0) = T_0(x)$$

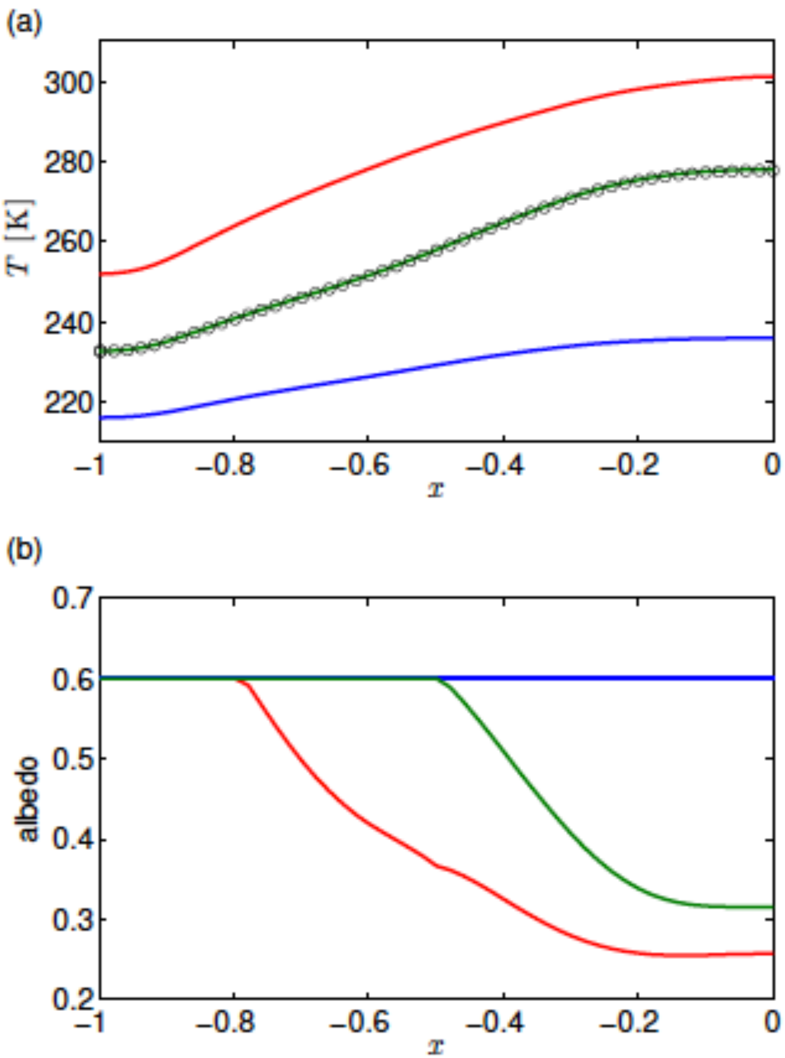
$$\boxed{k(x,T) = k_1(x) + k_2(x)g(T)}$$ Diffusion parameter

$$g(T) = c_4/T^2 \exp(-c_5/T)$$ Greenhouse effect

$$\boxed{\alpha(x,T) = \{b(x) - c_1[T_m + \min(T - c_2 z(x) - T_m, 0)]\}_c}$$

Ice-albedo feedback

Temperature Profiles

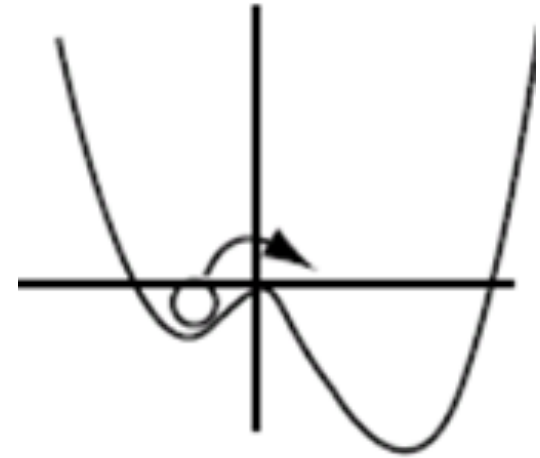


- Stable Climate (red): Warm - Attractor
- Stable Climate (blue): Snowball - Attractor
- Un-climate (green): In-between - Repellor

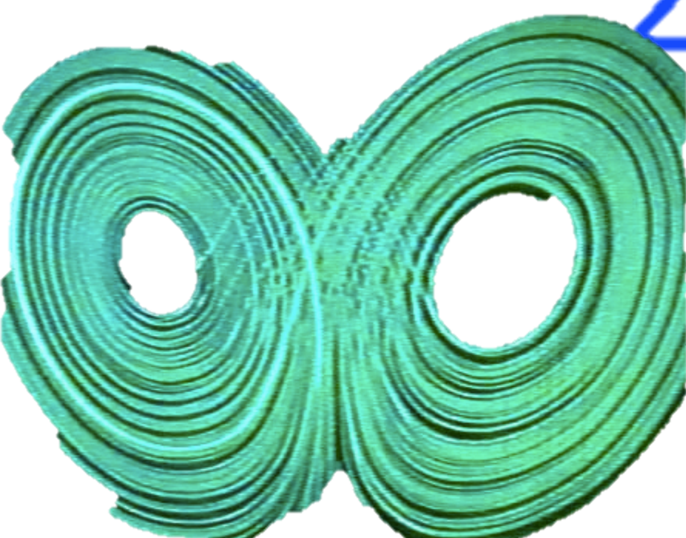
Ghil (1976)

Grebogi, Ott, Yorke (1983):

From this..



…
to this!



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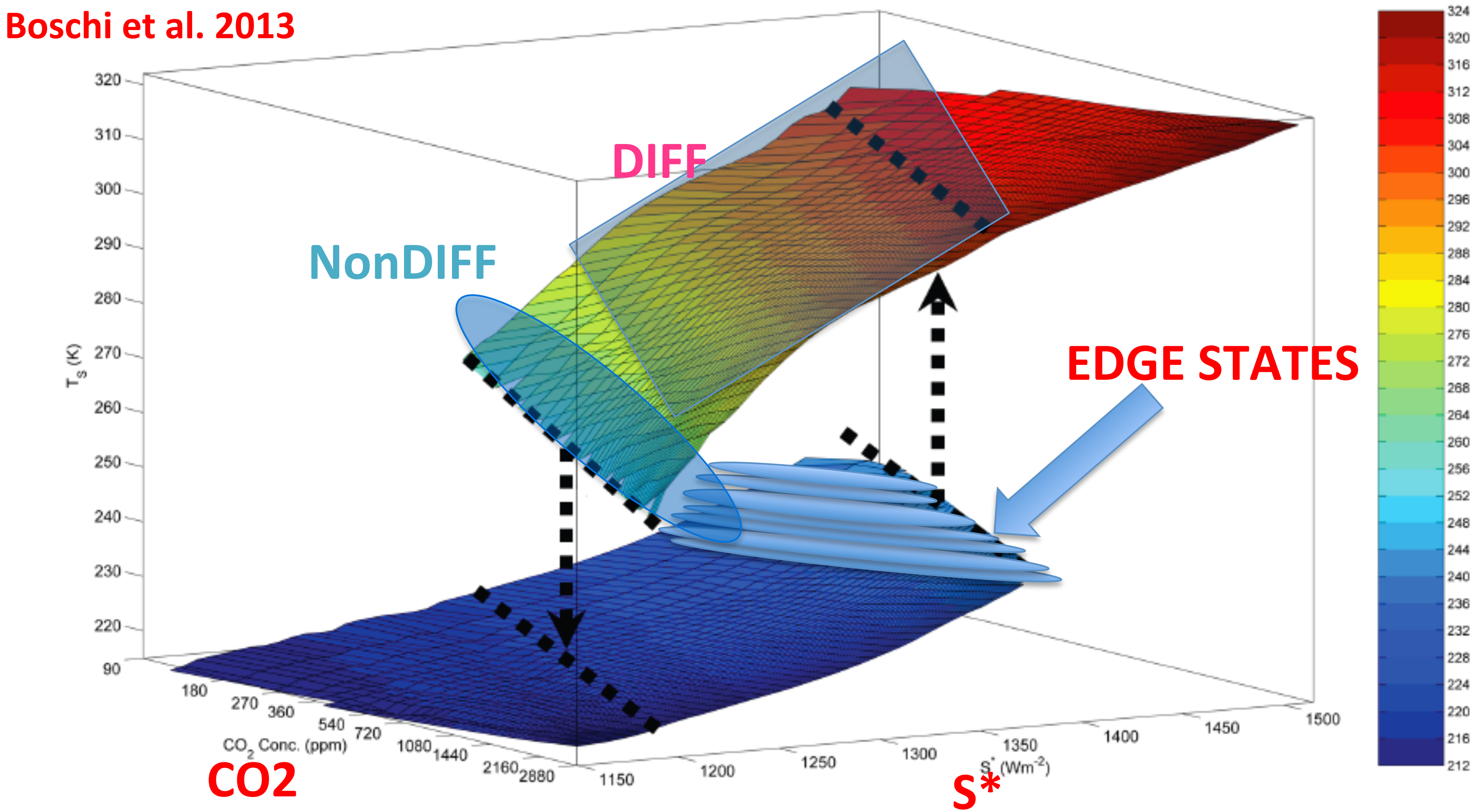
Climatic Edge States



Clim Dyn
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Clim. Dyn. 44, 3361-3381 (2015)

Global instability in the Ghil–Sellers model

Tamás Bódai · Valerio Lucarini · Frank Lunkeit ·
Robert Boschi

Received: 19 February 2014 / Accepted: 29 May 2014

Abstract The Ghil–Sellers model, a diffusive one-dimensional energy balance model of Earth's climate, features—for a considerable range of the parameter descriptive of the intensity of the incoming radiation—two stable climate states, where the bistability results from the celebrated ice-albedo feedback. The warm state is qualitatively similar to the present climate, while the cold state corresponds to snowball conditions. Additionally, in the region of bistability, one can find unstable climate states. We find such unstable states by applying for the first time in a geophysical nonequilibrium thermodynamics, the large scale temperature gradient between low and high latitudes. We find that a maximum of the temperature gradient is realized at the same value of the average temperature, about 270 K, largely independent of the strength of incoming solar radiation. Due to this maximum, a transient increase and nonmonotonic evolution of the temperature gradient is possible and not untypical. We also examine the structural properties of the system defined by bifurcation diagrams describing the equilibria depending on a system parameter

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Invited Article

Nonlinearity 30, R32-R66 (2017)

Edge states in the climate system: exploring global instabilities and critical transitions

Valerio Lucarini[1,2,3] and Tamás Bódai[1,2]

[1] Department of Mathematics and Statistics, University of Reading, Reading, United Kingdom
[2] Centre for the Mathematics of Planet Earth, University of Reading, Reading, United Kingdom
[3] CEN, Meteorological Institute, University of Hamburg, Hamburg, Germany

E-mail: v.lucarini@reading.ac.uk

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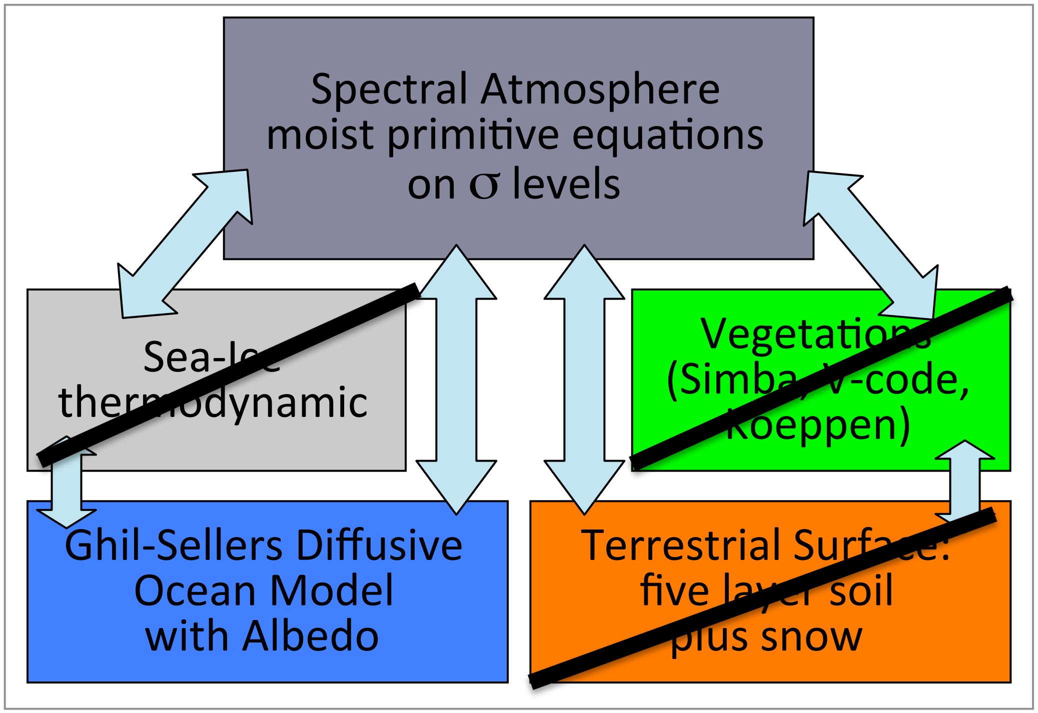
CrossMark

Recommended by Professor Bruno Eckhardt

Abstract
Multistability is a ubiquitous feature in systems of geophysical relevance and provides key challenges for our ability to predict a system's response to perturbations. Near critical transitions small causes can lead to large effects

PUMA-GS



- Simplified Climate Model
- Primitive Equations Atmosphere
- Simple Diffusive Ghil '76 Ocean
- Slow and fast climate variability
- Positive/negative feedbacks

O(10^5) d.o.f.

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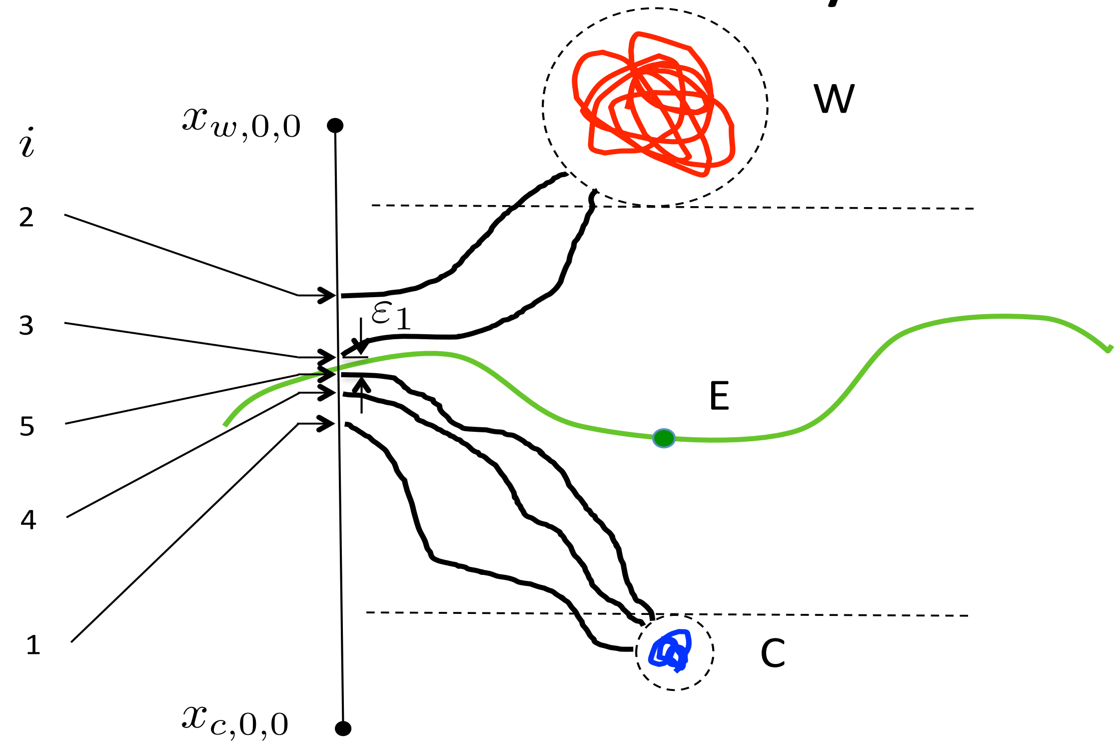
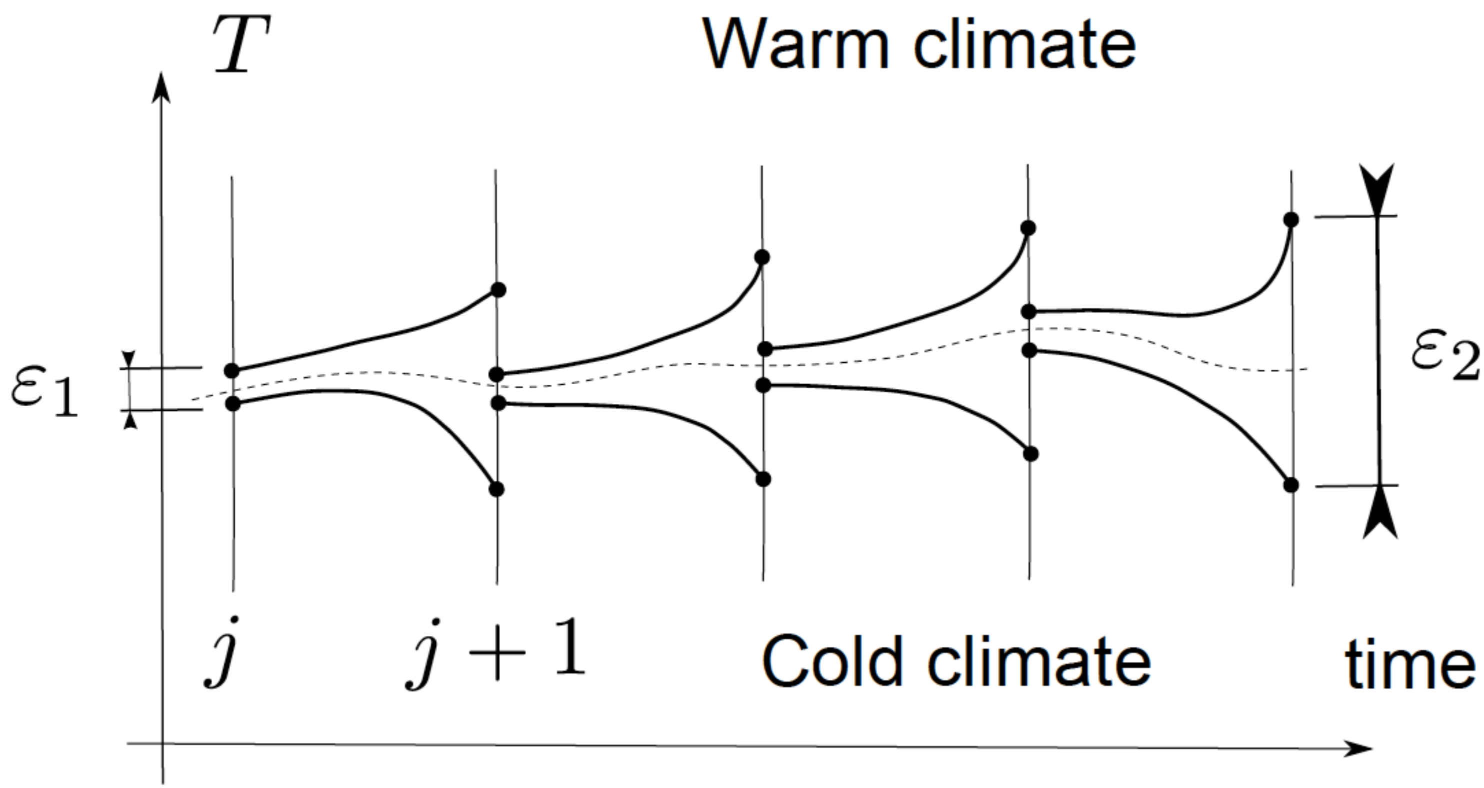
Edge State: “relative attractor” on the basin boundary

- After GOY 1983: Eckhardt et al., multistable fluids
  - Pipe flow, Plane Couette Flow with fixed point vs (transient) turbulent regime for suitable Re
- Dynamics on the basin boundary



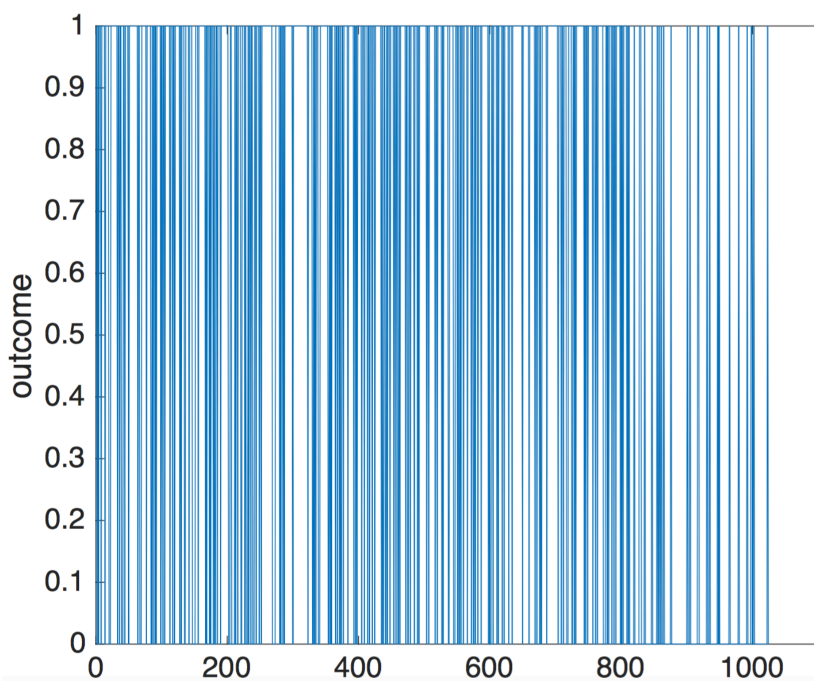
Tracking the Edge State

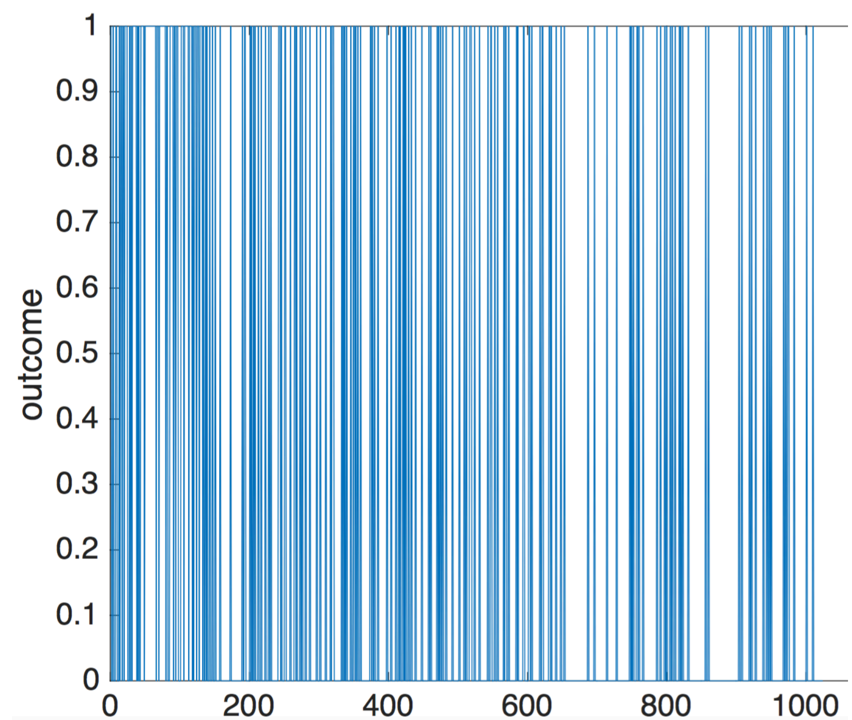


- Dynamics of an orbit near the the basin boundary
  - First, it relaxes VERY rapidly towards the edge state;
  - Second, it decides towards which attractor it should head to;
  - Third, it reaches the final destination.
- Reiterating the procedure we end up on the .. “Melancholia states” (after L. Von Trier’s movie)

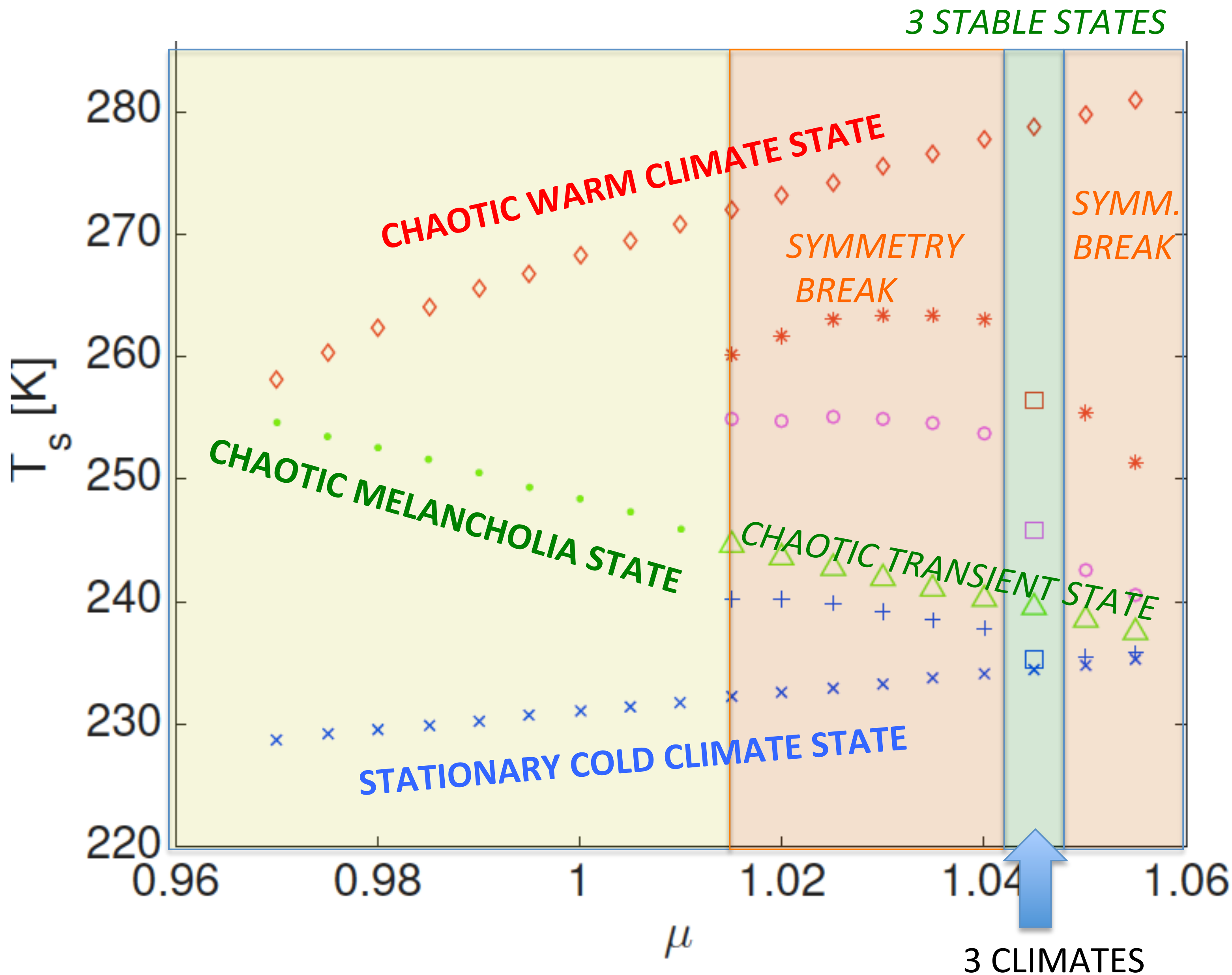
A closer look at the boundary

- Pikovsky: “is the basin boundary smooth?”
  - It is folded, indeed fractal.
  - Result of 1024 integrations between two trajectories near the boundary, 0.5 K difference in T_{surf}
  - Result of different time scales of instability on the edge states vs across it

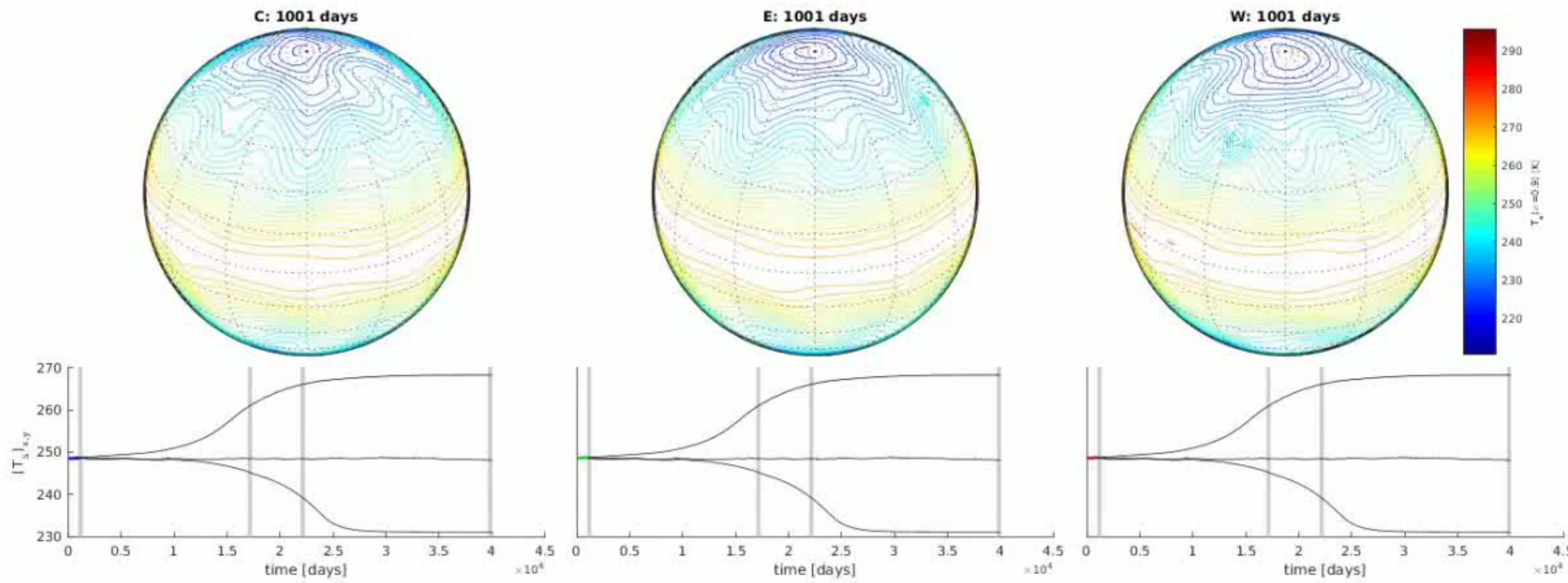




Multiple Steady States

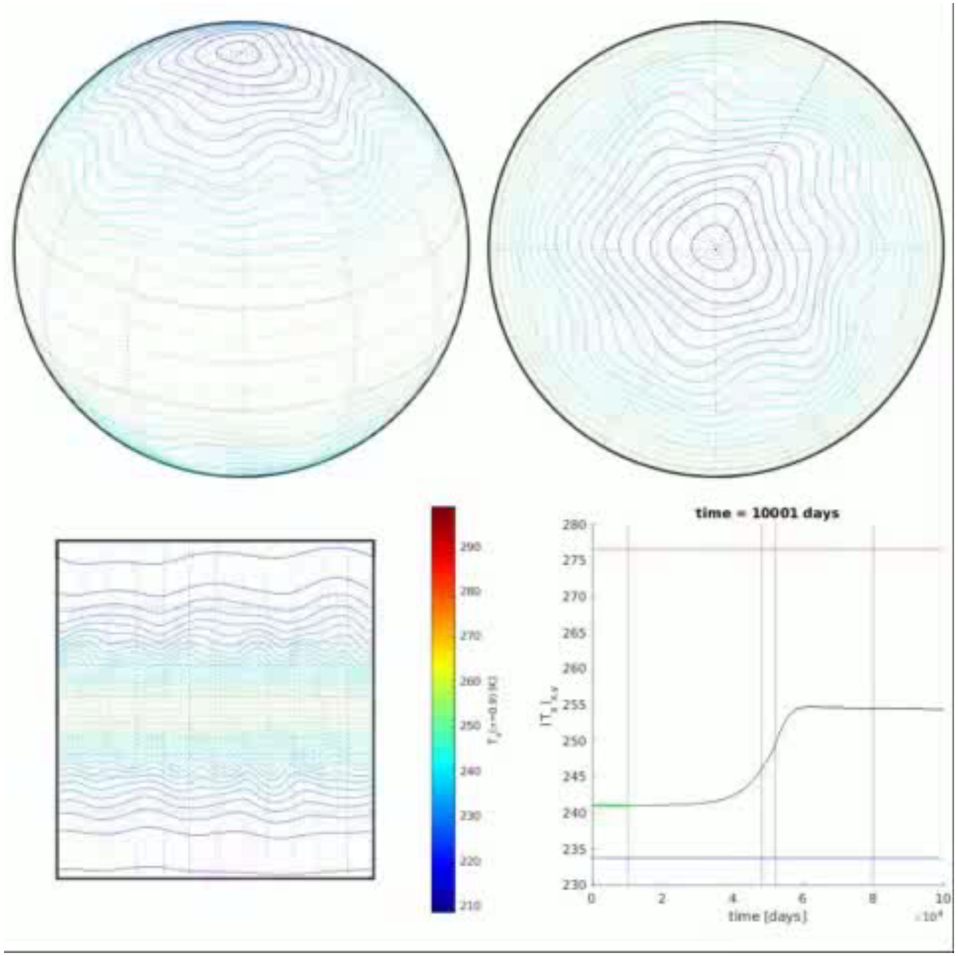


Multistability



Cold State Melancholia State Warm State

Symmetry Break of the Melancholia State



Conclusions

- Climate as nonequilibrium statistical mechanical system
- Beyond invariant measure: pullback attractor
- Response theory for smooth response
  - Predict climate change,
- High sensitivity and mixing rate
  - Transfer operator approach
- Multistability and Tipping points
  - Melancholia State, gate for the transitions
- We can construct the Melancholia state, which separates the warm from the snowball climate state, also in real GCMs
- The edge state is the gate allowing for noise-induced transitions between the two basins of attraction
- Note: Proximity to Tipping Points can be detected using EVT (Faranda, Lucarini, Manneville, 2014)

Beware Critical Transitions!