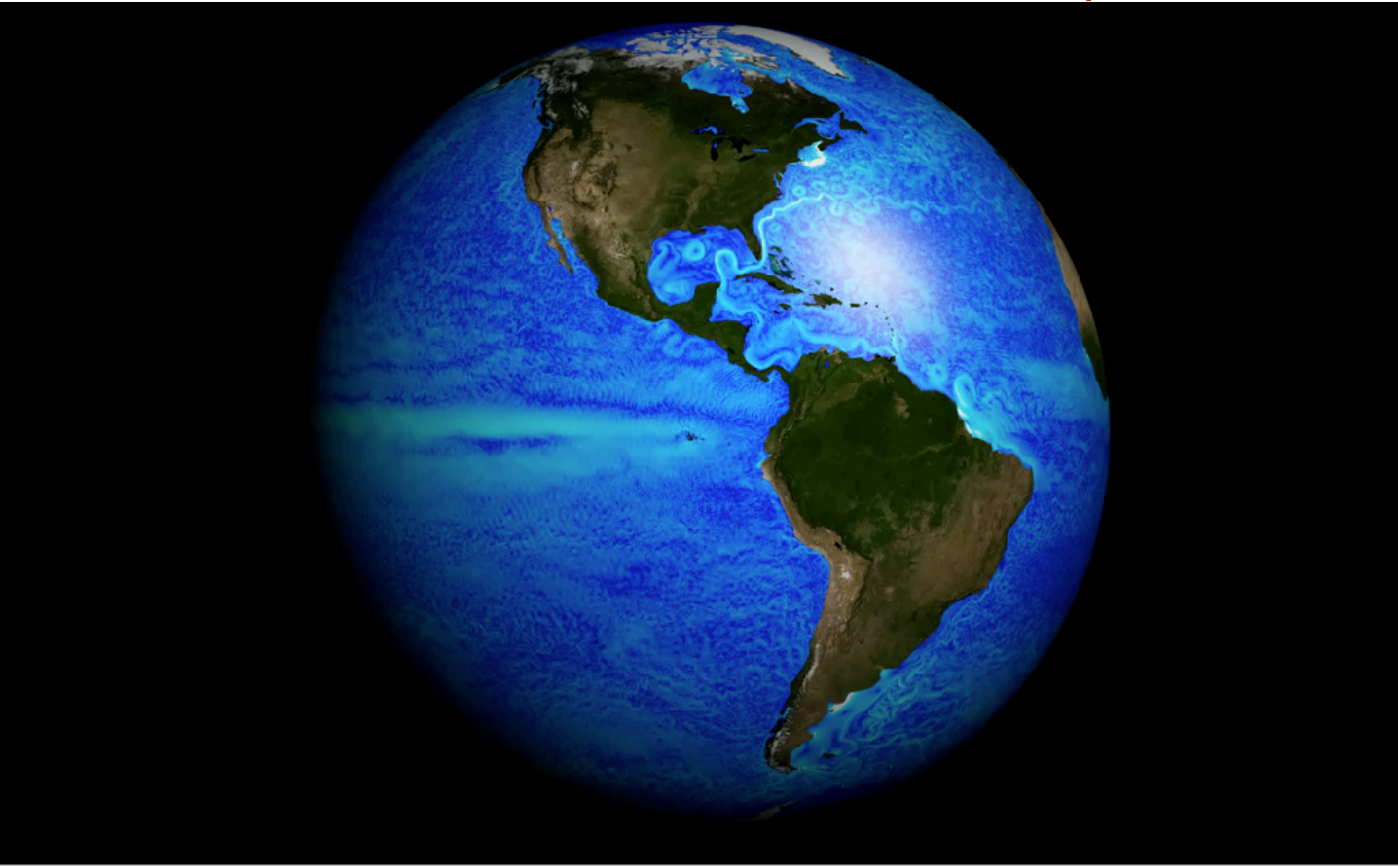
Tides, and a little about the energetics of ocean turbulence

Les Houches 2017
WRY, Lecture 1

Motion in the ocean — sea-surface speed

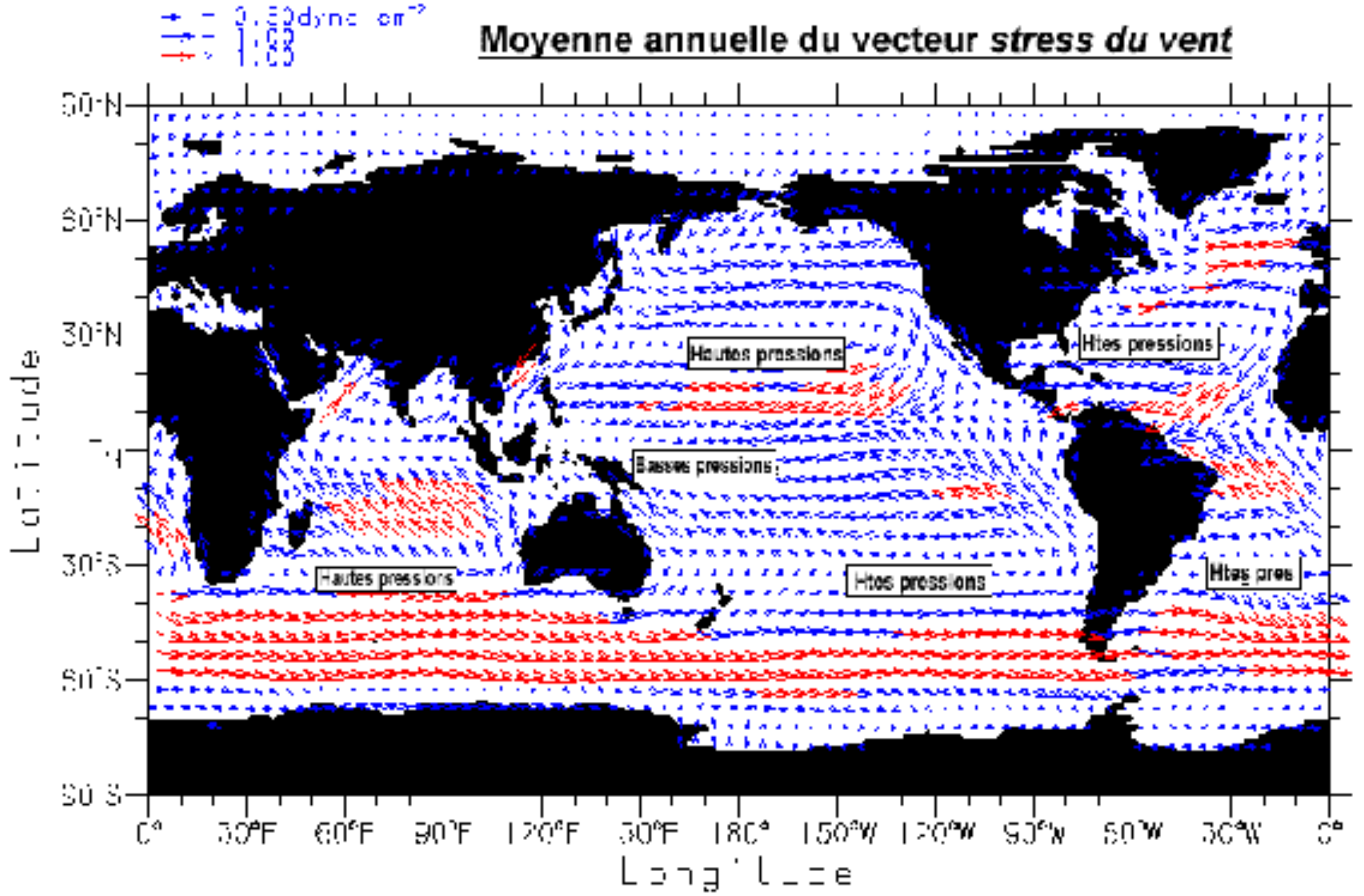


LLC4320 sea surface speed animation by C. Henze and D. Menemenlis (NASA)
1/48 degree, 90 vertical levels, MITgcm spun up from ECCO v4 state estimate.

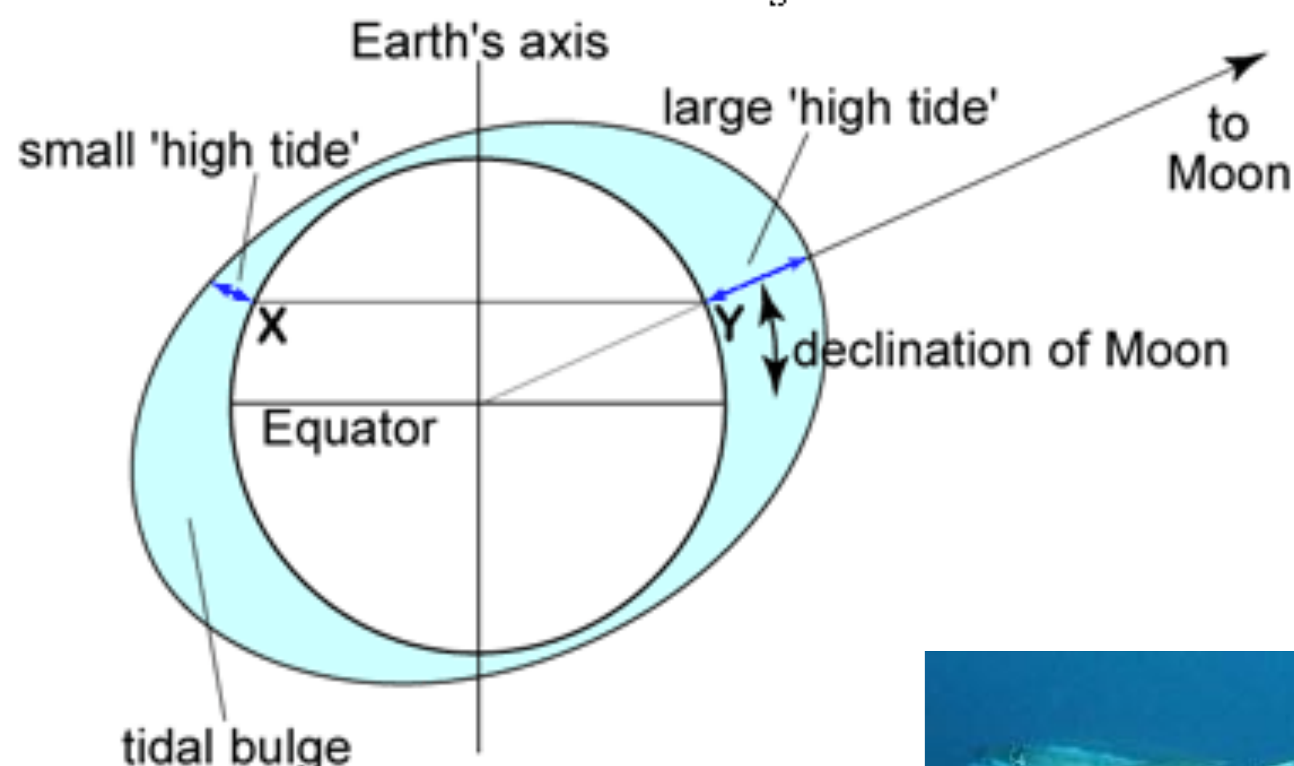
What are the main sources of ocean kinetic energy, turbulence and mixing?

We just saw the big two sources in the movie.

Wind stress — mechanical forcing at the sea surface.



Tides — extraction of energy from the spin of the Earth.



Bioturbation — fish wakes etc.

Geothermal heating at the ocean floor.

Nonuniform heating and cooling at the sea surface — Horizontal Convection.

A quantitative estimate

302 WUNSCH ■ FERRARI

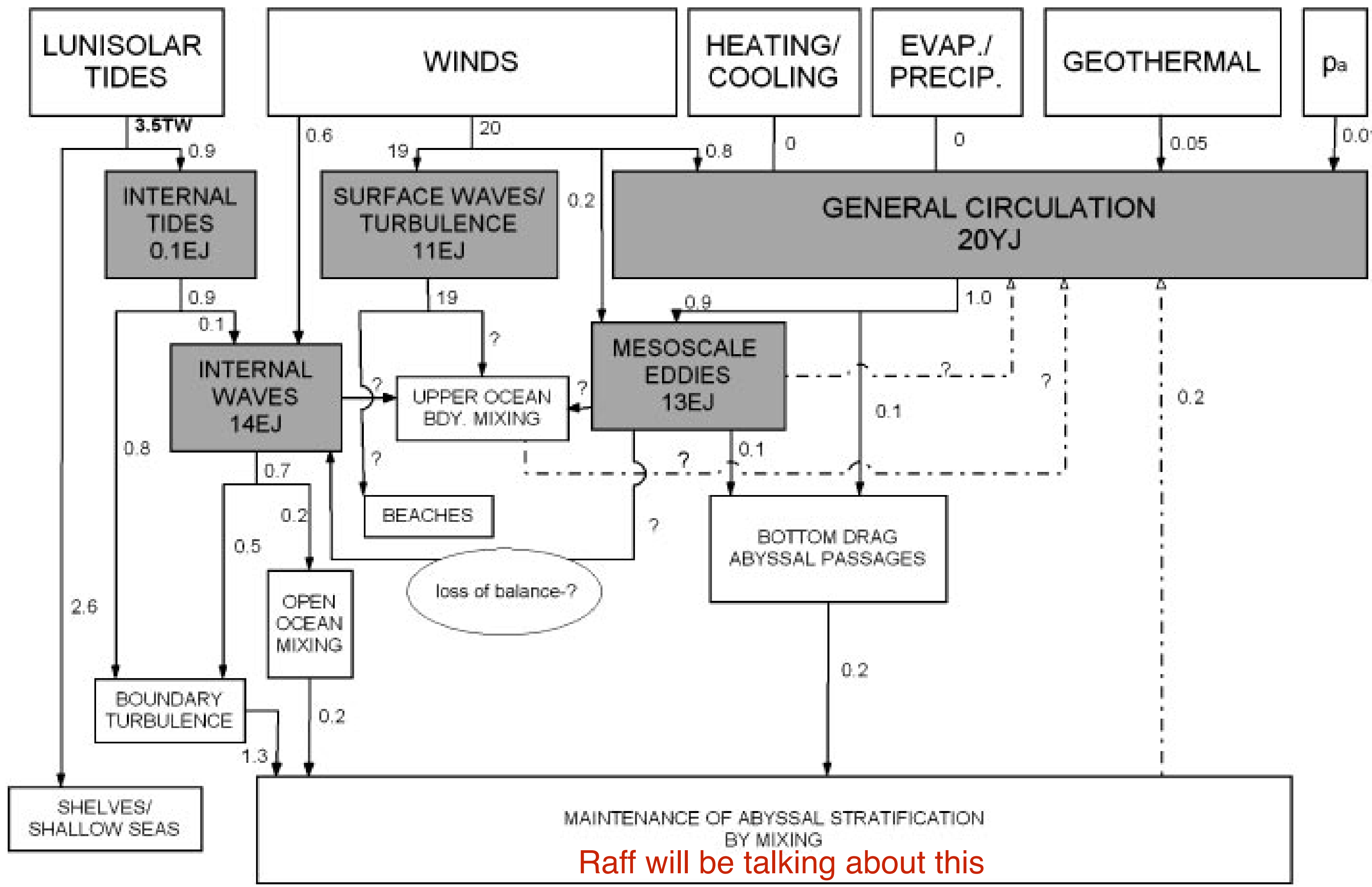
Fluxes are in TerraWatts (TW) = 10^{12}W

"All numbers, except 3.5TW, are uncertain by at least a factor of 2, and maybe 10."
(But some numbers are zero!)

No fishy forcing shown.

Note the unimportance of convection as an ocean energy source. (Lecture 2)

A large part of today's lecture is about tides and the 3.5TW.

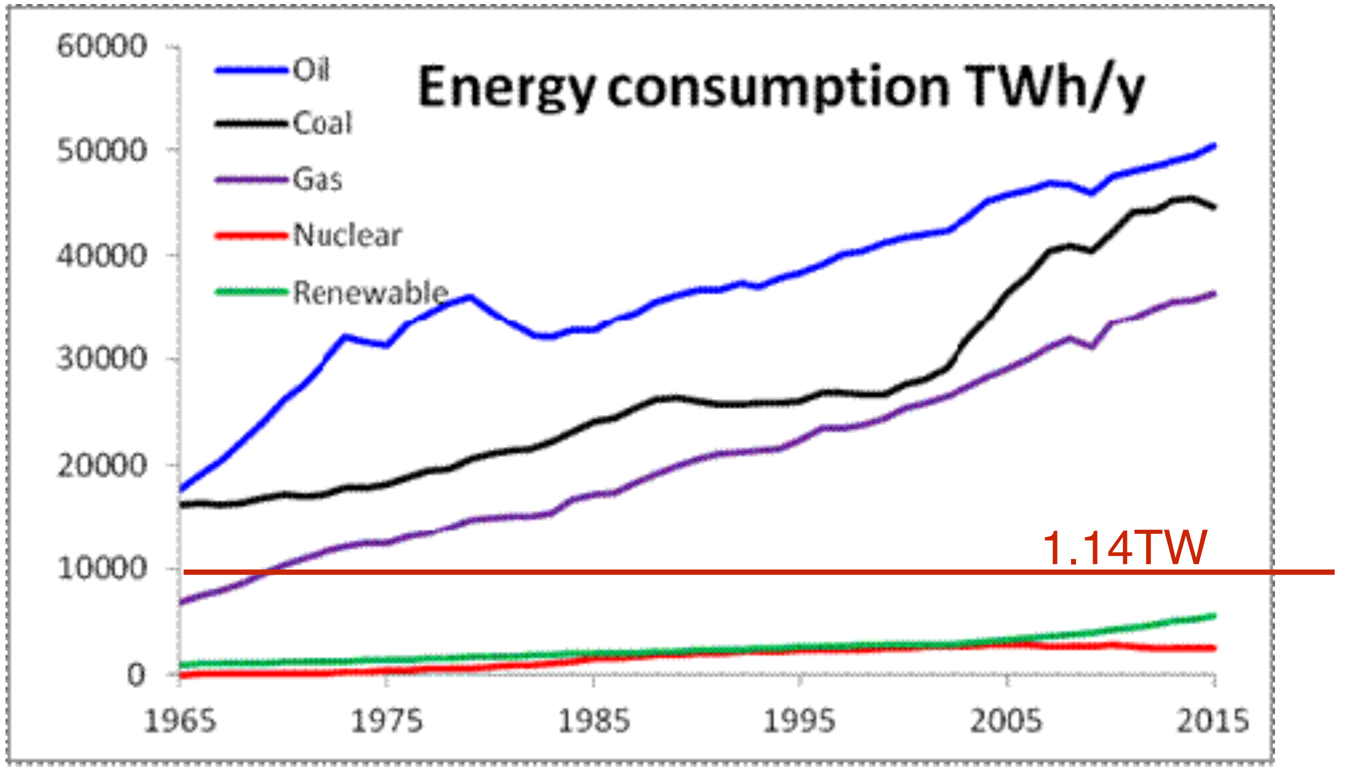


ExaJoules (EJ) = 10^{18}J
YottaJoules (YJ) = 10^{24}J

Putting 1TW into perspective

According to the IEA, human energy consumption in 2013 was 18TW. A nuclear power station is about 10^{-3}TW.

Solar radiation is 175,000TW and the ocean transports about 2000TW from equator-to-pole. Geothermal heat flux from the Earth's interior is 31TW.
(Exercise: why do W&F show 0.05TW?)



BP Statistical Review of World Energy

1TW is tiny by geophysical standards. But physical oceanographers insist that fluxes of this magnitude are important for ocean circulation.

$$\frac{1\mathrm{TW}}{\text{mass of the ocean} = 0.7\times 10^{21}\mathrm{kg}} = 1.4\times 10^{-9}\mathrm{Watts\ kg^{-1}}$$

A ballpark number for observed turbulent energy dissipation in most of the ocean:

$$\begin{aligned}\varepsilon^{\text{ocean}} &= 10^{-9}\ \mathrm{W\ kg^{-1}}\\ &= \text{a hairdryer in } 1\mathrm{km}^3 \text{ of seawater}\end{aligned}$$

You should remember this number….

Viscous dissipation, ε, is directly measured

Oakey (1982)

The dissipation rate is determined from the velocity-shear spectrum by integration and (including factors for isotropy) is given by

$$\epsilon = \frac{15}{2}\nu\overline{\left(\frac{\partial u'}{\partial z}\right)^2} \quad [\mathrm{m^2\,s^{-3}}]. \tag{1.3}$$

In this expression $\overline{(\partial u'/dz)^2}$ is the variance in the vertical gradient of turbulent fluctuations in the x direction (z down). We could also have used $\overline{(\partial v'/dz)^2}$.

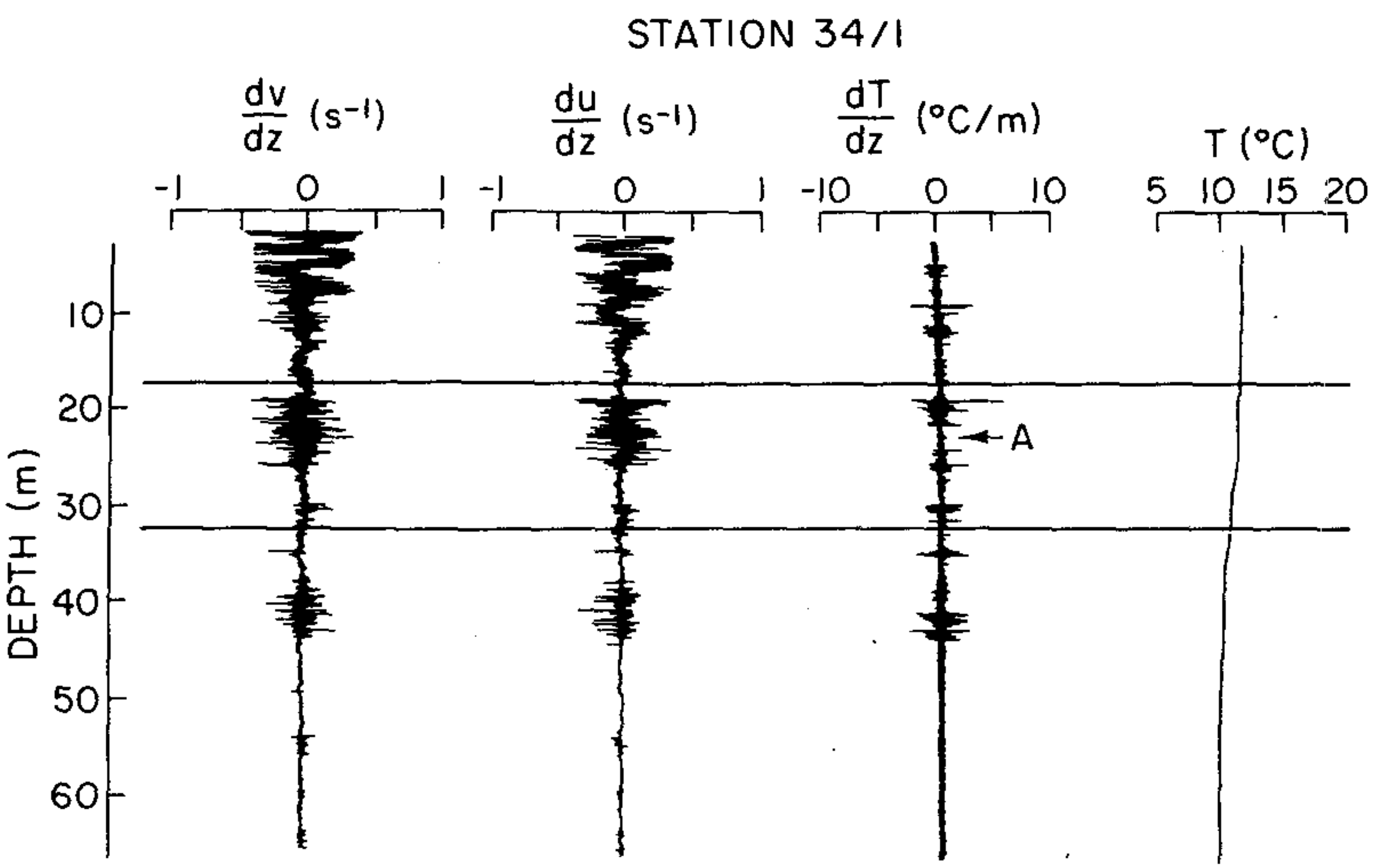


FIG. 1. A vertical profile (Station 34/1) showing velocity shear, temperature gradient and temperature. For the section, delineated by two horizontal bars, between 17 and 32 m, $\epsilon = 7 \times 10^{-8}\ \mathrm{m^2\,s^{-3}}$, $\chi_\theta = 1.9 \times 10^{-7}\ \mathrm{°C^2\,s^{-1}}$ and $dT/dz = 5.7 \times 10^{-2}\ \mathrm{°C\,m^{-1}}$.

Moum et al. (1995)

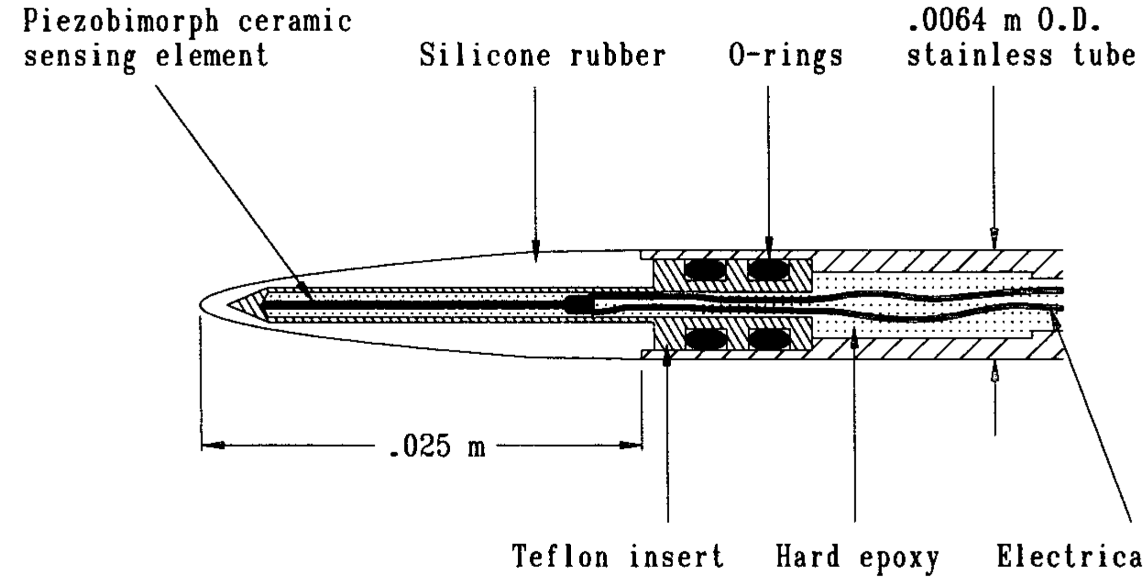
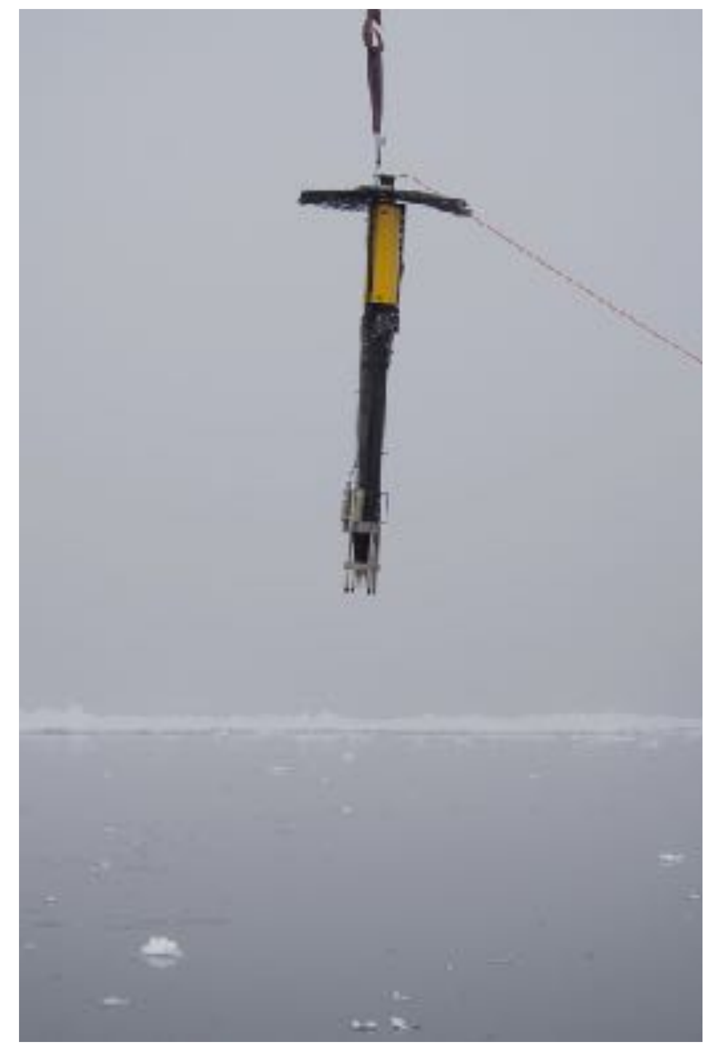


FIG. A2. Schematic of airfoil probe used by the Oregon State University group.



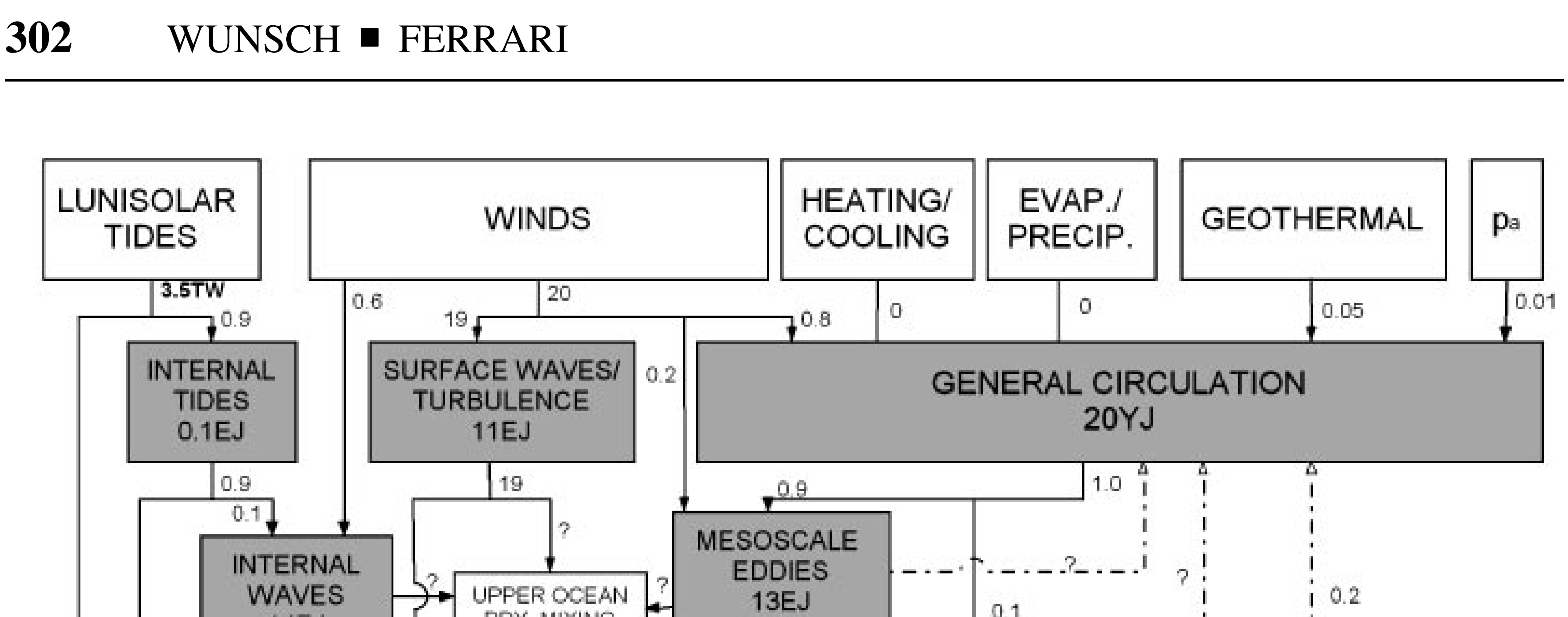
From next lecture:

$$\ell_{\mathrm{K}} = \left(\frac{\nu^3}{\varepsilon}\right)^{1/4} \sim 1\mathrm{cm}$$

Today's lecture is about the well know number: 3.5 TW of tidal energy input.

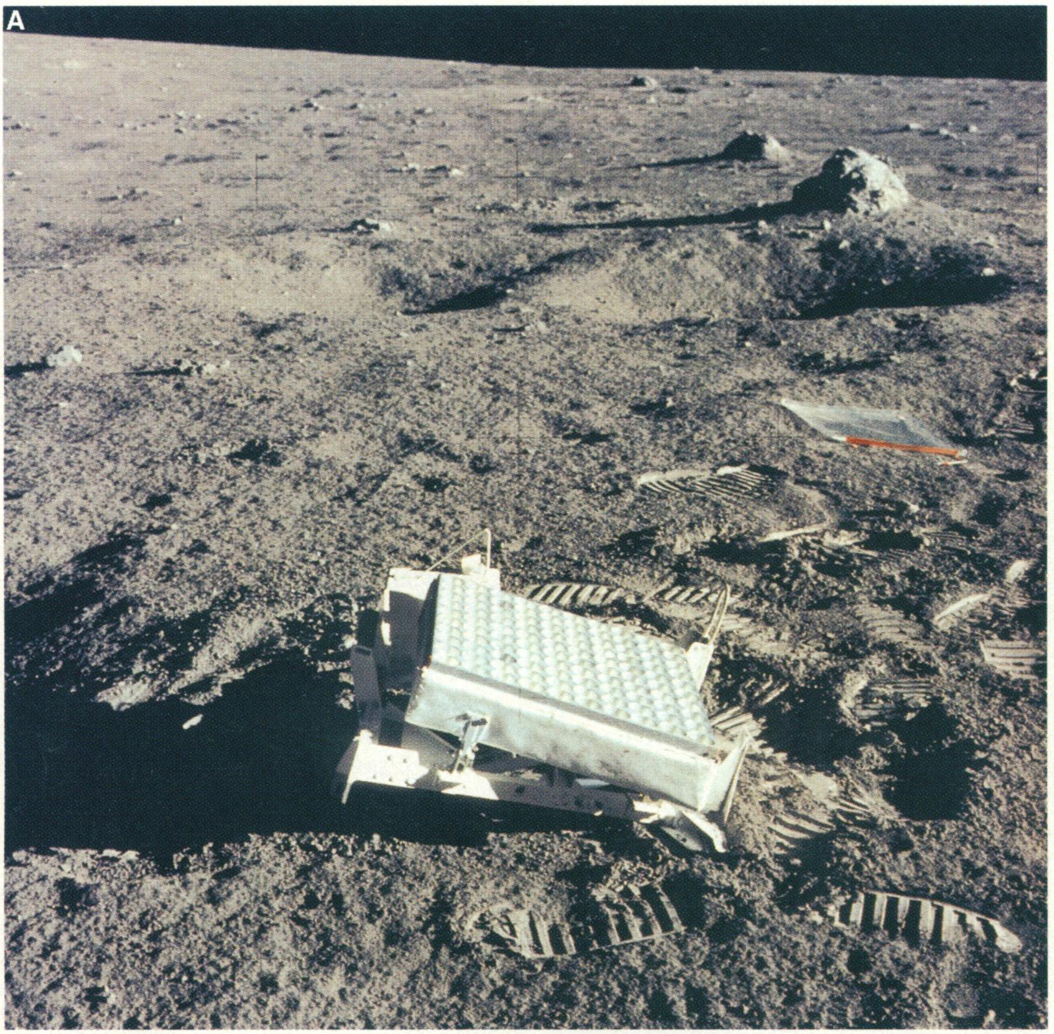
How is this small number so well known?

Because another small number is very well know.



Lunar laser ranging: lunar recession

Dickey et al. (1994)



LLR experiment left by Apollo 11 in 1969.

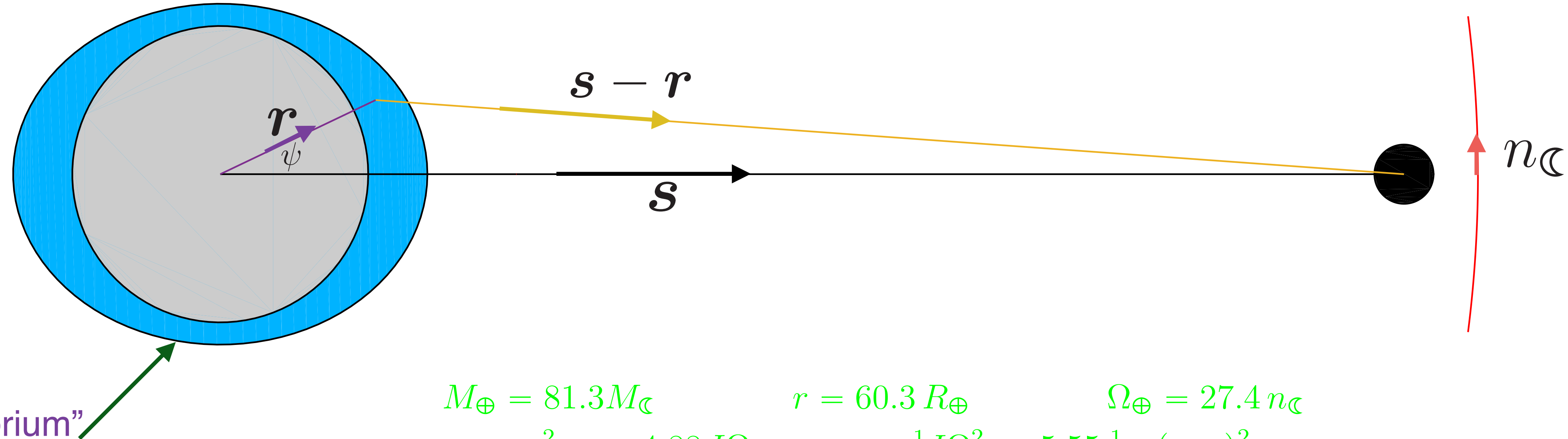
1 in 10^{17} photons are received back on Earth.

GSFC image

The moon is spiraling away from the Earth at 3.82±0.07cm/year.

The Moon is gaining orbital energy and the earth is losing spin energy. (The LOD is increasing.) The gains and loses are unequal: the lost spin energy, ~ 3.5TW, goes mainly into ocean tides. From the perspective of astronomy the 3.5TW is tidal friction. Oceanographers believe this is an important forcing on the ocean circulation.

“Ideal” tides — no friction, and the equilibrium tidal bulge



The “equilibrium” tidal bulge

$M_{\oplus} = 81.3 M_{\leftmoon}$ $\quad r = 60.3\, R_{\oplus}$ $\quad \Omega_{\oplus} = 27.4\, n_{\leftmoon}$

$m r^2 n_{\leftmoon} = 4.88\, I \Omega_{\oplus}$ $\quad \frac{1}{2} I \Omega_{\oplus}^2 = 5.55\, \frac{1}{2} m (n_{\leftmoon} r)^2$

The gravitational potential due to the moon at a field point ***r*** near the Earth

$$
\begin{aligned}
U &= -\frac{G M_{\leftmoon}}{|\boldsymbol{s} - \boldsymbol{r}|} \\
&\approx -\frac{G M_{\leftmoon}}{s} \left(1 + \frac{r}{s} \cos\psi + \left(\frac{r}{s}\right)^2 P_2(\cos\psi) \right)
\end{aligned}
$$

The “dipole” is balanced by the centripetal acceleration and the residual tidal potential is the “quadrupole”.

$$
U_2 = -\frac{G M_{\leftmoon}}{s^3} r^2 P_2(\cos\psi)
$$

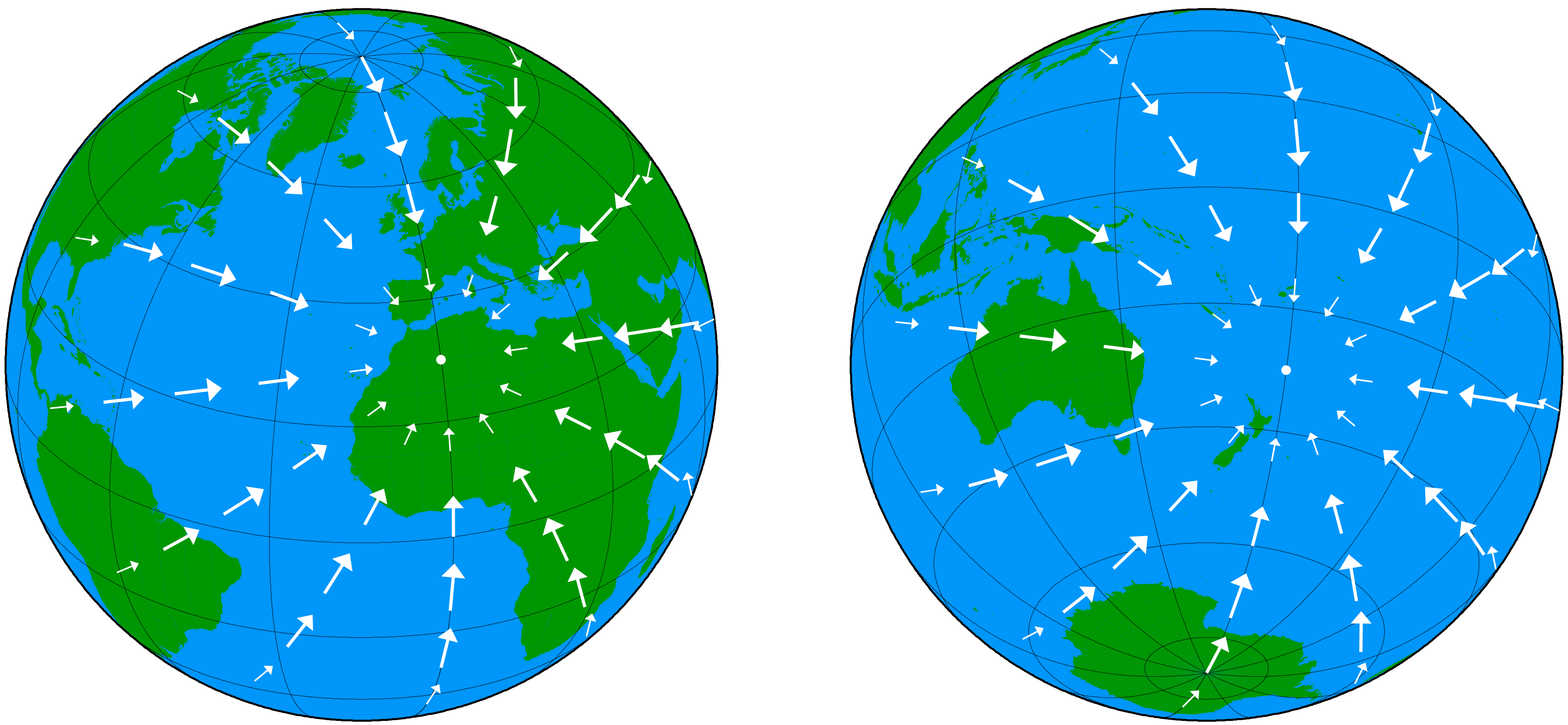
$P_2(\mu) = \frac{1}{2}(3\mu^2 - 1)$

The gravitational force of the moon is largely cancelled by the acceleration of the Earth's center. The residual "tidal force" varies over the extent of the ocean and has a depth-independent horizontal component.

$$\boldsymbol{F}_{\text{tide}} \sim \frac{GM_{☾}R_{\oplus}}{S^3} \sim 10^{-7}g$$

The sublunar point moves westward across the surface of the earth at ~450m/s.

$$\sqrt{gH} \sim \sqrt{10 \times 4000} = 200\text{m s}^{-1}$$



The horizontal component of the tidal force is directed towards the sublunar point (in the Sahara) and its antipode (North of New Zealand). As the Earth rotates the pattern moves westward.

Bjorn Gjevik

But the ocean does not follow the "equlibrium tidal bulge" because

(1) Continents get in the way.
(2) The ocean shallow-water wave speed is too slow.
(3) Friction between the ocean and the solid Earth.

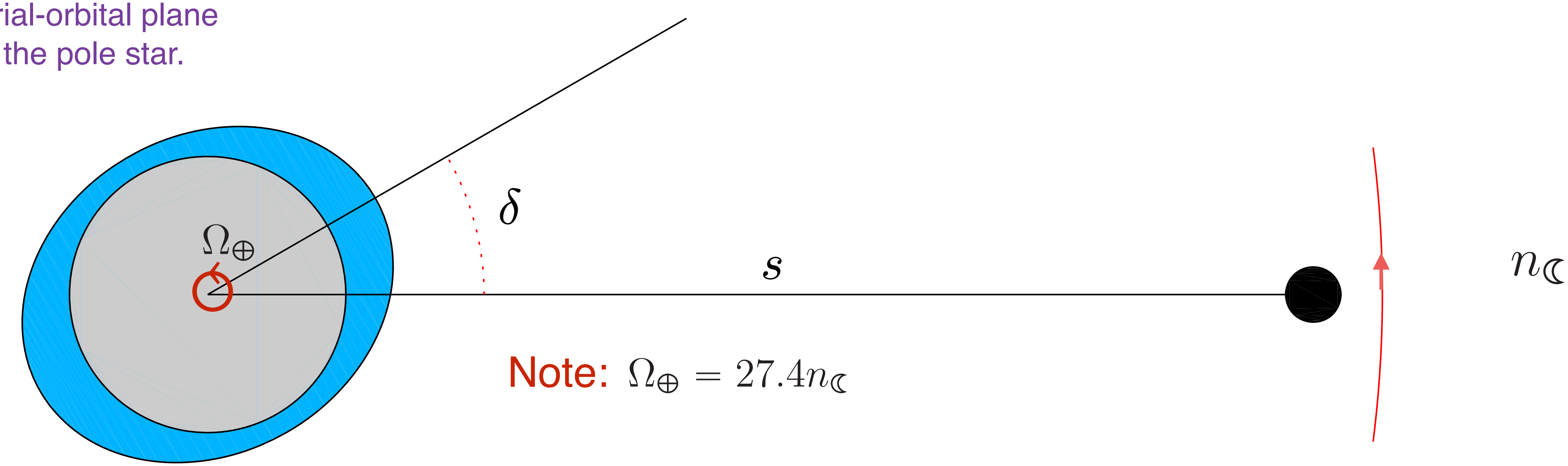
Even more complications

The tidally distorted solid Earth + Ocean, exerts a gravitational force back on the moon. Because of friction, this force "pulls" the moon along its orbit. This results in tidal torque.

I'll simplify the presentation by approximating the Lunar orbit so that forthwith (1) it is circular and (2) lies the equatorial plane of the Earth.

Friction: the tidal bulge of the Ocean is pulled in front of the Moon by the Earth's spin. Thus there is a torque pulling the moon along its orbit.

Looking at the equatorial-orbital plane from the pole star.



Note: $\Omega_\oplus = 27.4 n_{☾}$

If $\delta \neq 0$, angular momentum is transferred from the Earth to the Moon by a torque Γ.

$$I \frac{\mathrm{d}\Omega_\oplus}{\mathrm{d}t} = -\Gamma\,, \qquad \text{and} \qquad \underbrace{\frac{M_\oplus M_{☾}}{M_\oplus + M_{☾}}}_{m} \frac{\mathrm{d}}{\mathrm{d}t} s^2 n_{☾} = +\Gamma$$

We don't know the torque. But let's see what we can deduce from mechanics and the observed rate of lunar recession.

The present rate of lunar recession is $\dot{s} = 3.8\,\mathrm{cm\ yr^{-1}}$

Kepler's third law

$$s^3 n_☾^2 = G(M_☾ + M_\oplus)$$

Note the donkey effect of Lynden-Bell & Kalnajs (1972): "in azimuth stars behave like donkeys, slowing down when pulled forward and speeding up when held back".

$$\begin{aligned}\dot{n}_☾ &= -\frac{3}{2}\frac{\dot{s}}{s} n_☾ \\ &= -1.25 \times 10^{-23}\,\mathrm{rad\ s^{-2}} \\ &= -25.7\,\mathrm{arcseconds\ (century)^{-2}}\end{aligned}$$

The moon is accelerating, and slowing down.

The tidal torque

$$\begin{aligned}\Gamma &= m \frac{\mathrm{d}}{\mathrm{d}t} s^2 n_☾ \\ &= \frac{1}{2} m n_☾ s \dot{s} \\ &= 4.5 \times 10^{16}\ \mathrm{N\ m}\end{aligned}$$

The day is getting longer at 2.1 millisecond per century.

$$\begin{aligned}\Delta\mathrm{LOD} &= \frac{2\pi}{\Omega_\oplus^2} \frac{\mathrm{d}\Omega_\oplus}{\mathrm{d}t} \Delta t \\ &= \frac{2\pi}{\Omega_\oplus^2} \frac{\Gamma}{I} \Delta t\end{aligned}$$

$$m \stackrel{\mathrm{def}}{=} \frac{M_\oplus M_☾}{M_\oplus + M_☾}$$

Loss of energy from astronomy

More trickery with Kepler's third law.

$s^3 n_{☾}^2 = G(M_{☾} + M_{\oplus})$

$$\begin{aligned} E &= \tfrac{1}{2} I \Omega_{\oplus}^2 + \tfrac{1}{2} m s^2 n_{☾}^2 - \frac{G M_{☾} M_{\oplus}}{s} \\ &= \tfrac{1}{2} I \Omega_{\oplus}^2 - \tfrac{1}{2} m s^2 n_{☾}^2 \end{aligned}$$

The rate of change of astronomical energy is then

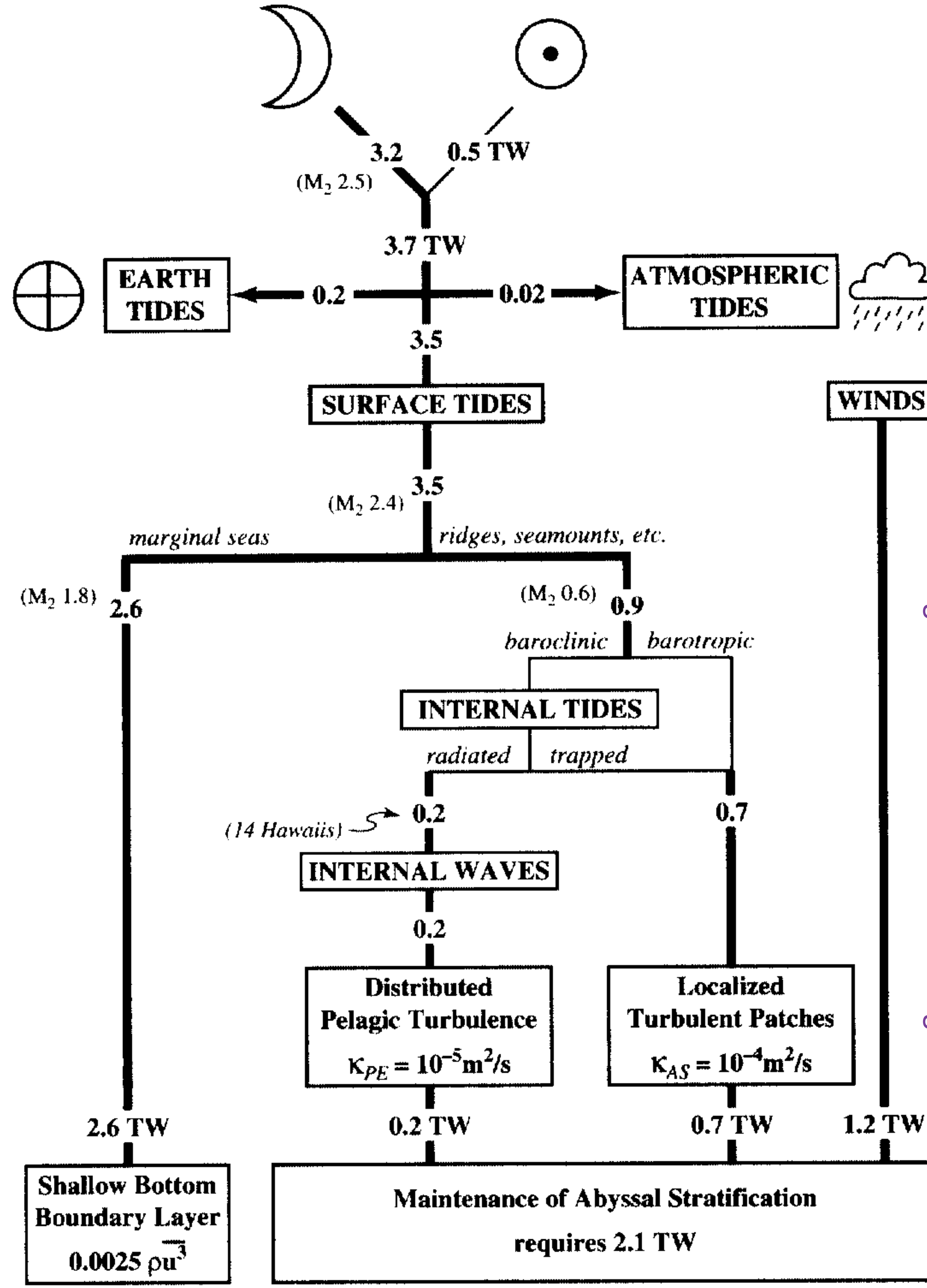
$$\begin{aligned} \frac{\mathrm{d}E}{\mathrm{d}t} &= -\left(\Omega_{\oplus} - n_{☾}\right) \underbrace{\tfrac{1}{2} m n_{☾} s \dot{s}}_{\Gamma} \\ &= -3.14 \times 10^{12}\mathrm{W} \end{aligned}$$

To check the above, recall that

$$I \frac{\mathrm{d}\Omega_{\oplus}}{\mathrm{d}t} = -\Gamma \,, \qquad \text{and} \qquad \underbrace{\frac{M_{\oplus} M_{☾}}{M_{\oplus} + M_{☾}}}_{m} \frac{\mathrm{d}}{\mathrm{d}t} s^2 n_{☾} = +\Gamma$$

What happens to lost orbital energy?

3.14TW becomes 3.7TW when the solar tides are included. But there is also some dissipation in the solid Earth.)



From Munk & Wunsch (1998)
Energetics of Tidal and wind mixing.

The time scale of tidal dissipation

A ballpark estimate of ocean tidal kinetic energy

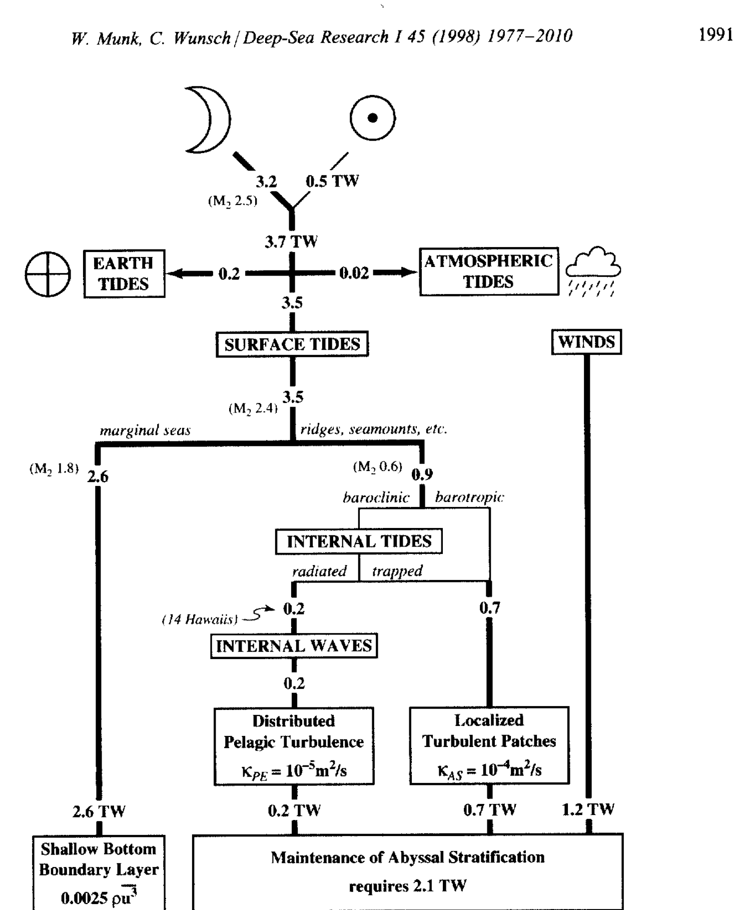
$$KE = \frac{1}{2} \times \underbrace{7 \times 10^{20}}_{\text{ocean mass}} \times (\underbrace{0.04}_{\text{RMS tidal velocity}})^2$$
$$= 8.75 \times 10^{17}\text{J}$$

The implied "renewal time" for tidal energy is a few days.

$$T = \frac{8.75 \times 10^{17}\text{J}}{3.7 \times 10^{12}\text{W}} = 2.7\text{days}$$

Munk (1997) says 2 days.

Where and how does this tidal dissipation occur? We'll return to this question at the end of the lecture. First an aside.

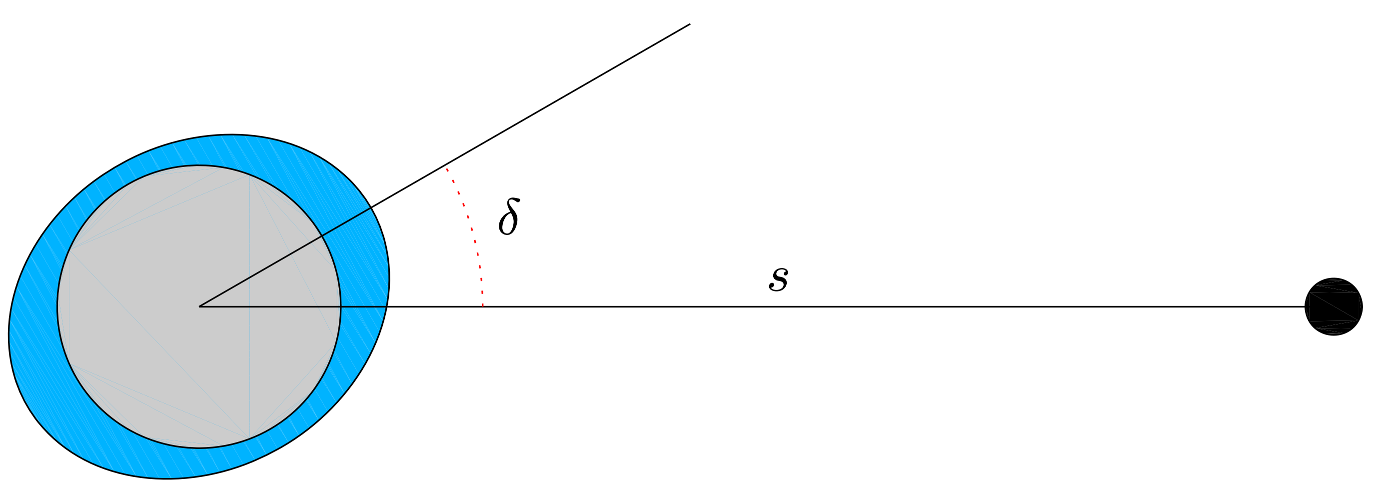


Munk & Wunsch (1998)

History of the lunar orbit and the present anomalous high dissipation

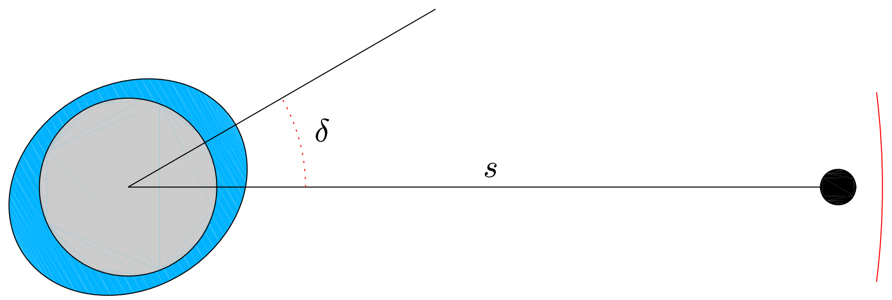
"Men will always aspire to peer into the remote past to the utmost of their power, and the fact that their success or failure cannot appreciably influence their life on Earth will never deter them from such endeavors." — G. H. Darwin, *The Tides*

To "peer into the remote past", we must model the variation of the torque Γ. The tidal potential of the Moon "bulges" the Solid Earth + Ocean. The gravity of the bulge then acts on the Moon. As a consequence, Γ depends very strongly on the Earth-Moon separation s.



$$\Gamma \propto s^{-6}$$

The response of the Earth + Ocean to the lunar tidal forcing is a "bulge", with its own gravity field.



$$U^{☾} = -\frac{GM_{☾}}{s^3} r^2 P_2(\cos\psi) + \cdots$$

Solid Earth + Ocean

This framework is well justified for solid-Earth tides. We apply it here to the ocean+solid Earth.

Augustus Edward Hough Love

This is a definition of k_2 and δ.

$$U^{\text{bulge}}|_{@R_\oplus} = -k_2 \frac{GM_{☾}}{s^3} R_\oplus^2 P_2\left[\cos(\psi - \delta)\right] + \cdots$$

The Love number
$k_2 = 0.245$

But the tidal bulge is not exactly along the E-M line of centers
$\delta = 2.9^\circ$

Caveats

This framework is well justified only for solid-body tides. We apply it to the ocean+solid Earth.

Love's formula

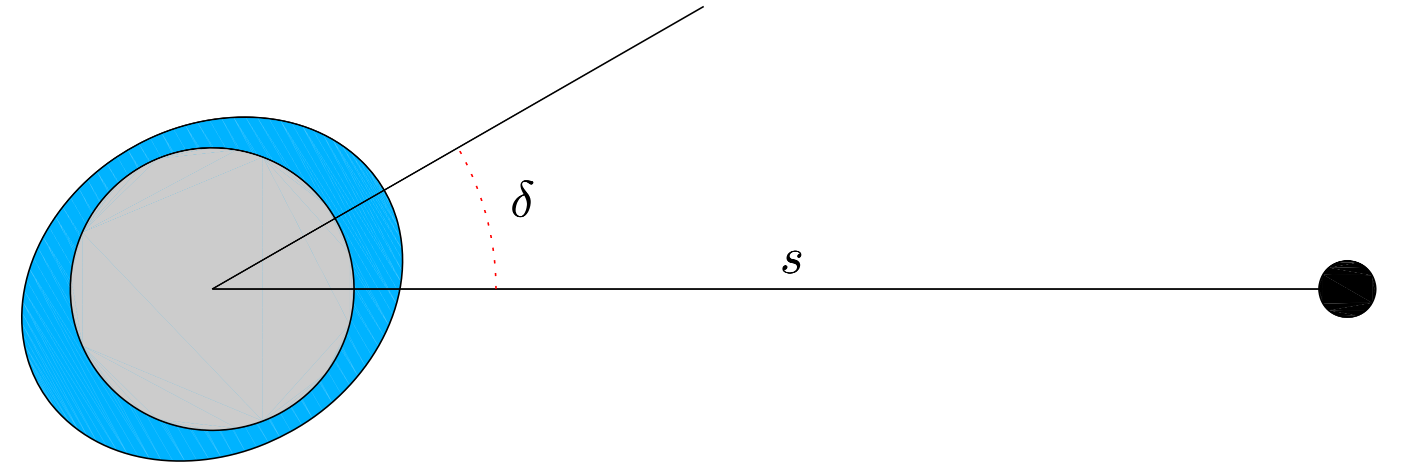
$$k_2 = \frac{3\, g\rho R_\oplus}{2g\rho R_\oplus + 19\mu}$$

$$\mu = \text{elastic rigidity}$$

There are higher multipole terms in all gravitational potentials — but these involve larger inverse powers of *s*. Drop these terms.

This linear model won't be reliable if the Moon is close to Earth e.g., 10 Earth radii.

$$\Gamma \propto s^{-6}$$



At the surface
of the Earth

$$U^{\text{bulge}}|_{@R_\oplus} = -k_2 \frac{GM_☾}{s^3} R_\oplus^2 P_2\left[\cos(\psi - \delta)\right] + \cdots$$

Therefore the
external potential of
the bulge is
$r > R_\oplus$

$$U_{\text{bulge}} = -k_2 \frac{GM_☾}{s^3} R_\oplus^2 P_2\left[\cos(\psi - \delta)\right] \left(\frac{R_\oplus}{r}\right)^3$$

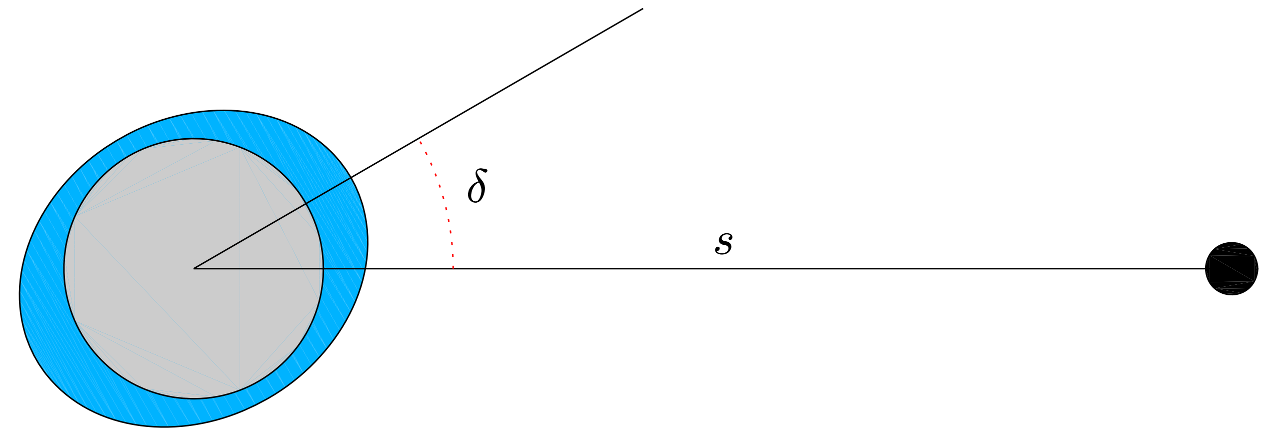
Evaluate the force, r^{-4},and
the torque, exerted on the
moon ($r=s$, $\psi=0$) by the
Earth+Ocean bulge.

$$\Gamma = \frac{3k_2 G M_☾^2 R_\oplus^5}{s^6} \sin\delta \cos\delta$$

A "business-as-usual" theory: constant δ

$$\Gamma = \frac{3k_2 G M_{\leftmoon}^2 R_\oplus^5}{s^6} \sin\delta\cos\delta$$

Most ignorance is in δ. Assume δ is constant and determined by LLR.



Combine with Kepler's third law: $s^3 n_{\leftmoon}^2 = G(M_{\leftmoon} + M_\oplus)$

$$\left.\begin{aligned} s^3 n_{\leftmoon}^2 &= G(M_{\leftmoon} + M_\oplus) \\ \frac{\mathrm{d}}{\mathrm{d}t}\frac{M_\oplus M_{\leftmoon}}{M_\oplus + M_{\leftmoon}} s^2 n_{\leftmoon} &= \gamma s^{-6} \end{aligned}\right\} \Rightarrow \boxed{\dot{s} = \dot{s}_0 \left(\frac{s_0}{s}\right)^{11/2}}$$

The solution is:

$$s = s_0 \left(1 + \frac{13}{2}\frac{\dot{s}_0}{s_0} t\right)^{2/13}$$

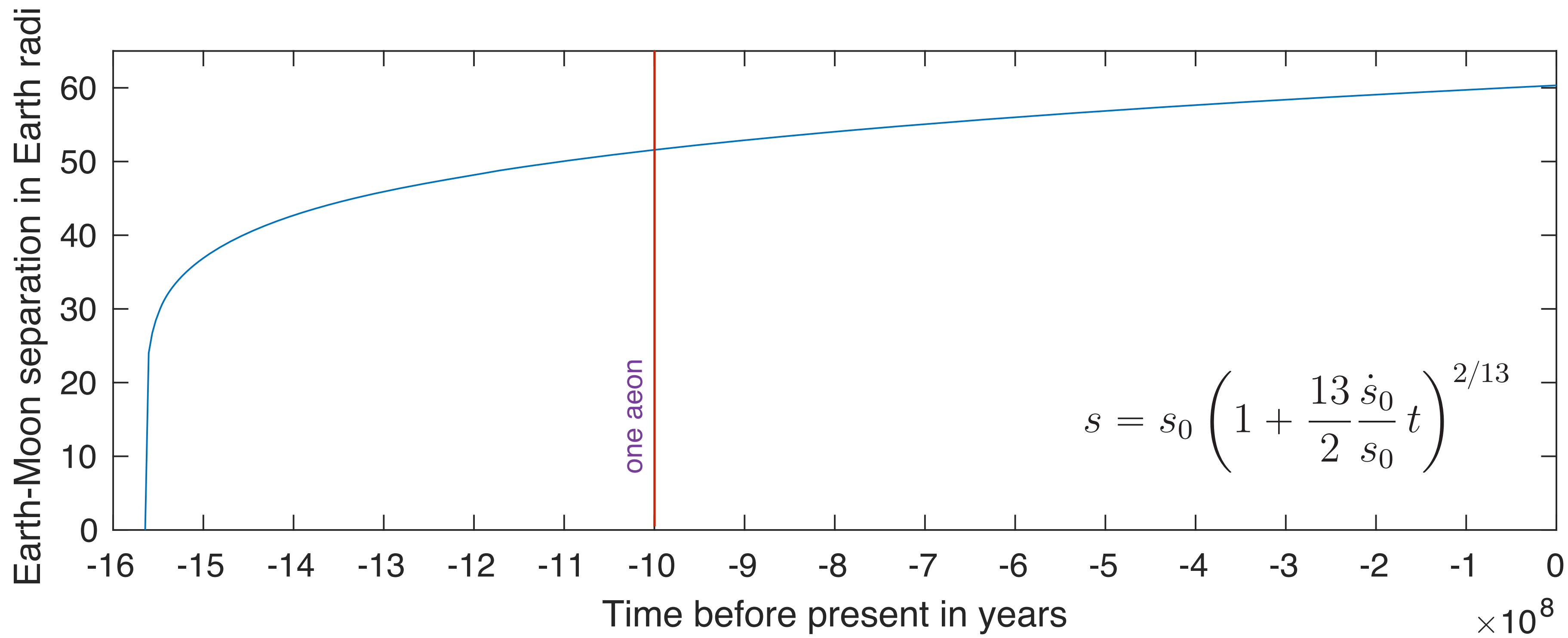
The time scale of lunar recession.

$$T^{\leftmoon} = \frac{2}{13}\frac{s_0}{\dot{s}_0} = 1.56 \times 10^9 \text{years}$$

More elaborate orbital calculations do not substantially alter the time scale

MacDonald (1964), perhaps Darwin?

The Gerstenkorn event



This 1.5 billion-year time scale for evolution of the lunar orbit is too short.

Oceans have grown over geologic history. But the sedimentary record does not indicate much change in the last two aeons.

At ~3 Earth radii the moon would fragment. At 6 or 7 Earth radii the ground temperature would be over 1000K and the oceans would evaporate. But well before that happened, the continents would be scoured by steaming 1km high tidal bores following the Moon on a 7 hour polar orbit.

None of these events occurred in the last 3 billion years (or probably ever). There are sedimentary structures indicative gentle deposition by lunar tides over 3 billion years ago.

Conclusion: present day tidal dissipation (i.e., the angle δ) is anomalously large.

Munk (1968)
"Once again – Tidal Friction"

Sedimentary Geology for Dummies

George E. Williams, “Earth’s Precambrian Rotation” (2000)

One must find a river delta with strong tidal flow that is also sheltered from surface wave activity. Sediment brought into the delta by rivers is carried into deeper water by ebb tides and deposited as a discrete layer. Each ebb tide deposits a single layer about one centimeter thick, but thicker during Spring tides and thinner during Neaps. Compaction produces layered siltstone with different layer thicknesses recording the lunar tidal cycle. Annual cycles with seasons are also reflected in layer thickness (e.g., more sediment delivered during the rainy season).

Studying the layers, one determines the number of days in a lunar month, and the number of months in a year.

A fundamental assumption is that the duration of a year does not change i.e., no transfer of angular momentum between the Earth and the Sun………

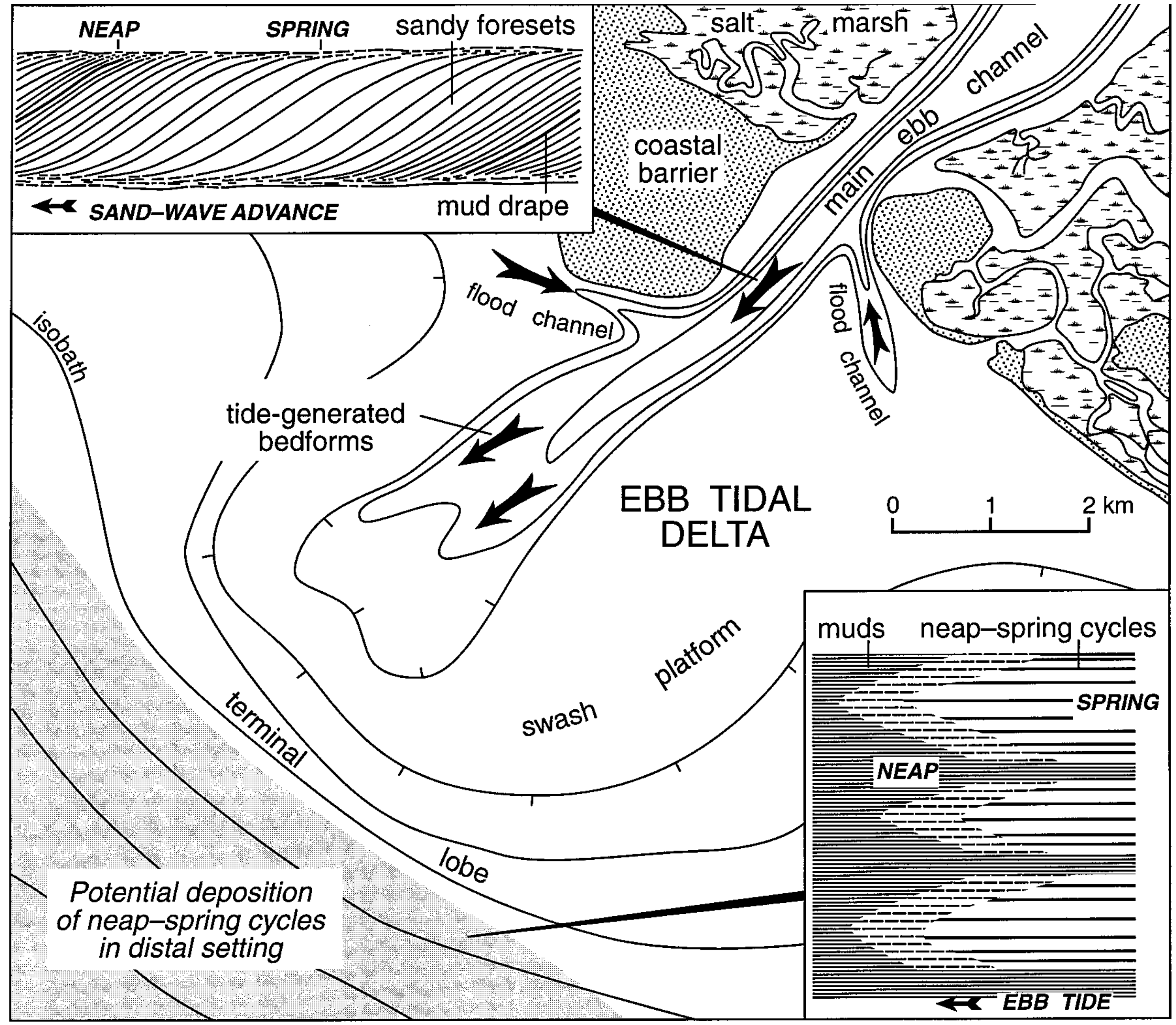


Figure 5. Envisaged environment of deposition for the Elatina-Reynella cyclic tidal rhythmites, employing a hypothetical ebb tidal delta adapted from *Imperato et al*. [1988]. The flood tides converge radially toward the tidal inlet, where fine-grained sediment is entrained by ebb tidal currents and transported mainly in suspension via the main ebb channel to deeper water offshore. There the suspended sediment settles to form neap-spring cycles of thin, graded laminae mostly of sand and silt (shown schematically in bottom inset). Farther offshore, the neap-spring cycles become progressively more abbreviated near positions of neap tides and eventually pass into marine shelf mud. Where protected from wave action, the distal ebb tidal delta setting favors the deposition and preservation of long rhythmite records. Tidal “bundle” deposits of cross-bedded sand (top inset) are confined to proximal, nearshore tidal channels. Modified from *Williams* [1989c] and reproduced with the permission of the International Union of Geological Sciences.

Sedimentary rhythmites

Periodic variations in the thickness laminae are a proxy for the tidal influence on sedimentation.

There are also biological proxies (sequential growth layers in corals). But the sedimentary record is better.

Conclusion: 620Ma ago the LOD was 21.9±0.4h, and there were 401 days/yr. Lunar recession was 2.17±0.31cm/yr.

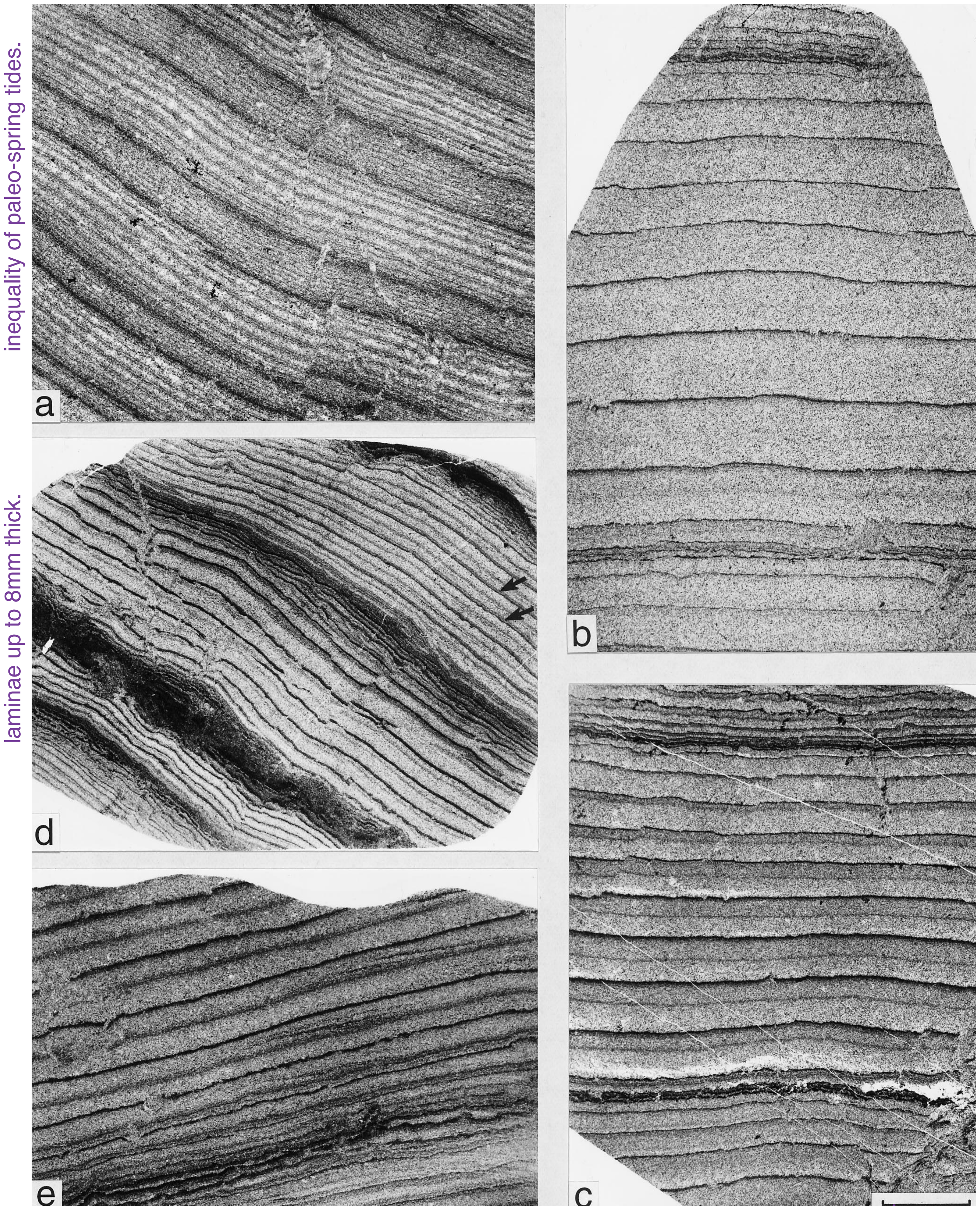
2450Ma ago the LOD was ~19h and there were 457 days/yr.

These changes in LOD are much less than those predicted by the constant-δ theory.

Reynella siltstone from South Australia.

Nine alternately thick and thin neap-spring cycles, indicating the monthly inequality of paleo-spring tides.

One complete neap-spring cycle containing diurnal laminae up to 8mm thick.



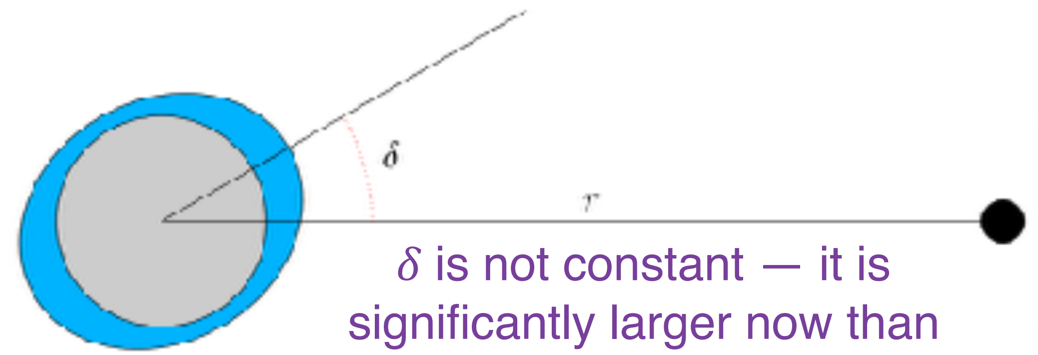
George E. Williams, “Earth’s Precambrian Rotation” (2000)

Why is present tidal dissipation so large?

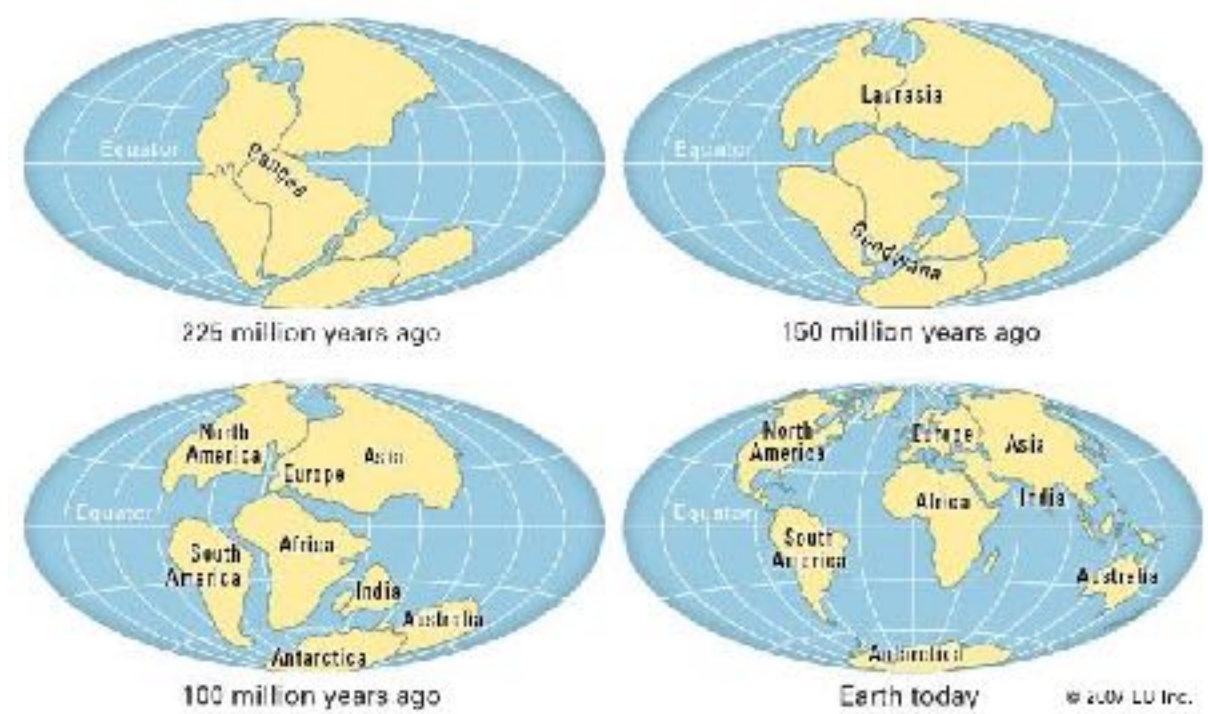
(1) It's not large — creation is more recent than you think.

(2) Perhaps G is changing on the billion year time scale? Probably not…

(3) " ... unless the present estimates for the accelerations are vastly in error, only a variable energy sink can solve the time-scale problem and the only energy sink that can vary significantly with time is the ocean." (Lambeck, 1980, page 288).



δ is not constant — it is significantly larger now than over geologic time.

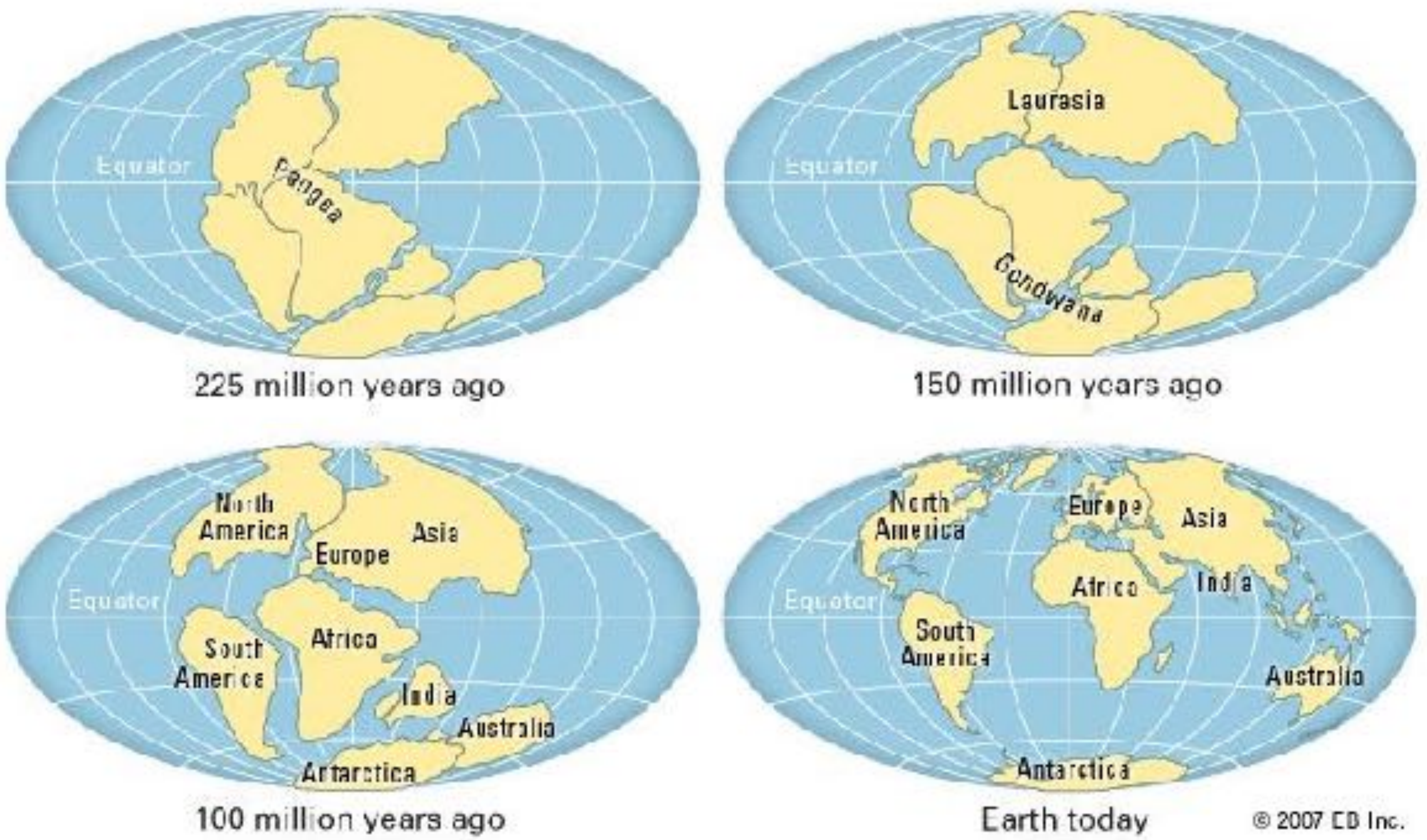


In addition to continental drift, the Coriolis parameter is increasing as the LOD decreases.

Continental drift, by itself, is not as important as one might guess

The resonant frequencies of ocean normal modes depend on the geometry of continents.

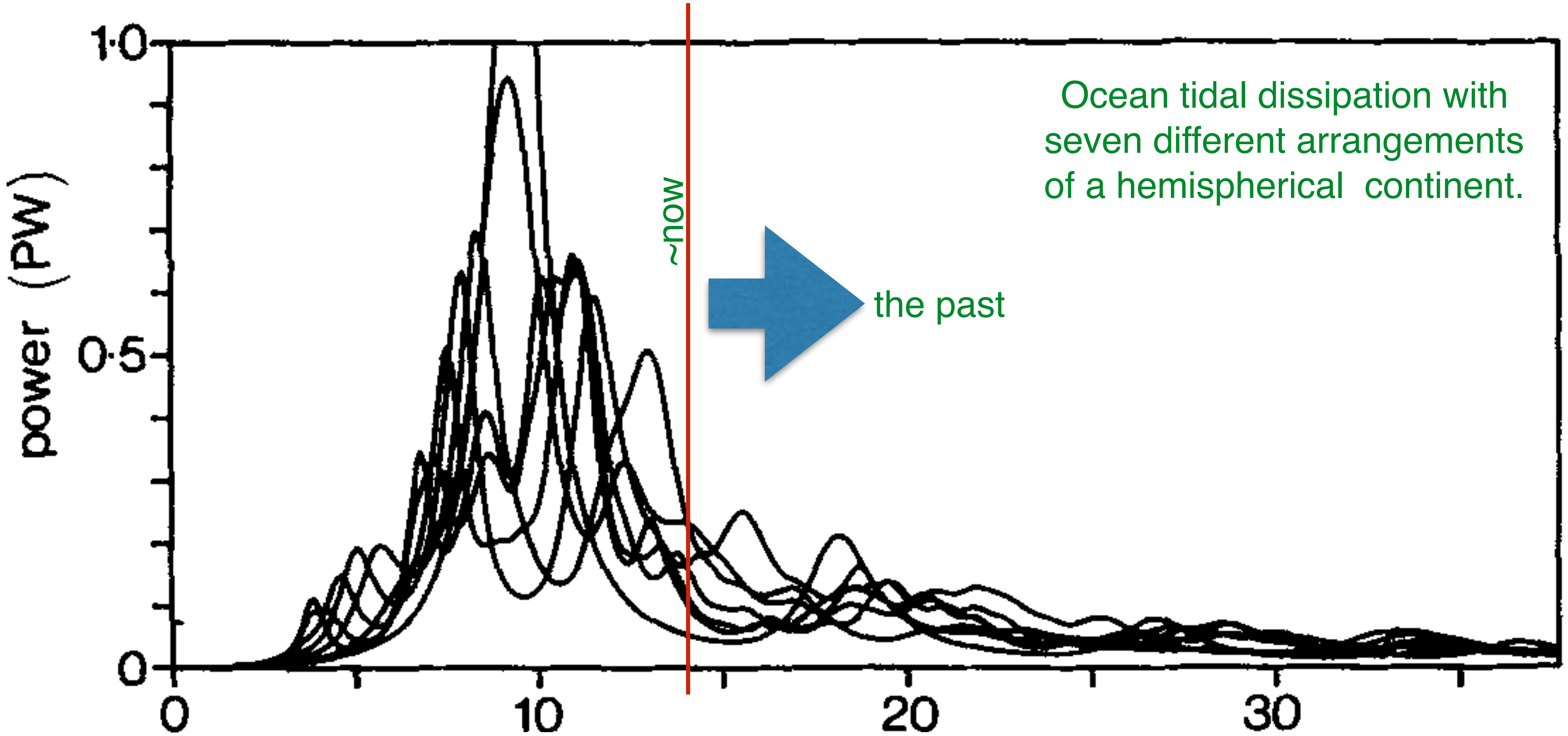
But the geometry changes on the fast time scale of 100 Myr. On the orbital evolution time scale, one can average over different geometries. This decreases the long-term effect of strong, but ephemeral, resonances.



The Coriolis parameter, however, is steadily increasing (and the LOD decreasing) as we go further back in time.

The systematic increase in the Coriolis parameter results in ocean normal modes with smaller length scales in the deep past

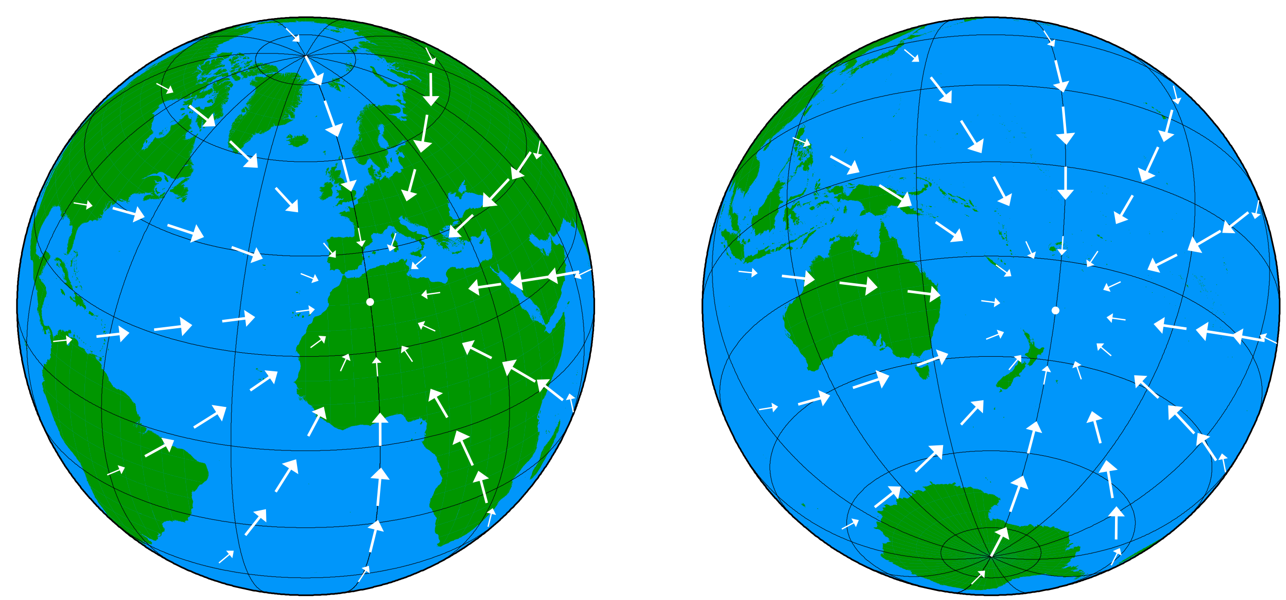
D.J. Webb, "tides and the evolution of the earth moon system" (1982)



Angular velocity of the tides rad/day
(currently ~4π rad/day)

Hypothesis: In the distant past, the smaller length scale of the modes is less well matched to the planetary-scale tidal forcing.

The mismatch between between large-scale forcing and the small scale of distant-past modes controls the ocean's effect on tidal friction.



End of the lunar-orbit aside

There is not much recent work on the problem of ocean normal modes, deep-time paleo-tides, and Earth-Moon tidal dissipation. None of the older work is highly cited.

REVIEWS OF GEOPHYSICS AND SPACE PHYSICS, VOL. 20, NO. 3, PAGES 457–480, AUGUST 1982

Secular Effects of Oceanic Tidal Dissipation on the Moon's Orbit and the Earth's Rotation

KIRK S. HANSEN[1]

Department of Geophysical Sciences, The University of Chicago, Chicago, Illinois 60637

83 citations

GEOPHYSICAL RESEARCH LETTERS, VOL. 26, NO. 19, PAGES 3045-3048, OCTOBER 1, 1999

Lunar Orbital Evolution: A Synthesis of Recent Results

Bruce G. Bills[1,2] and Richard D. Ray[3]

60 citations

ADVANCES IN GEOPHYSICS, VOL. 38

1996

DISSIPATION OF TIDAL ENERGY, PALEOTIDES, AND EVOLUTION OF THE EARTH–MOON SYSTEM

BORIS A. KAGAN

P. P. Shirshov Institute of Oceanology
Russian Academy of Sciences, St. Petersburg Branch
St. Petersburg, Russia

36 citations

JÜRGEN SÜNDERMANN

Institut für Meereskunde
Universität Hamburg
Hamburg, Germany

Geophys. J. R. astr. Soc. (1982) **70**, 261–271

Tides and the evolution of the Earth–Moon system

49 citations

D. J. Webb *Institute of Oceanographic Sciences, Wormley, Godalming GU8 5UB*

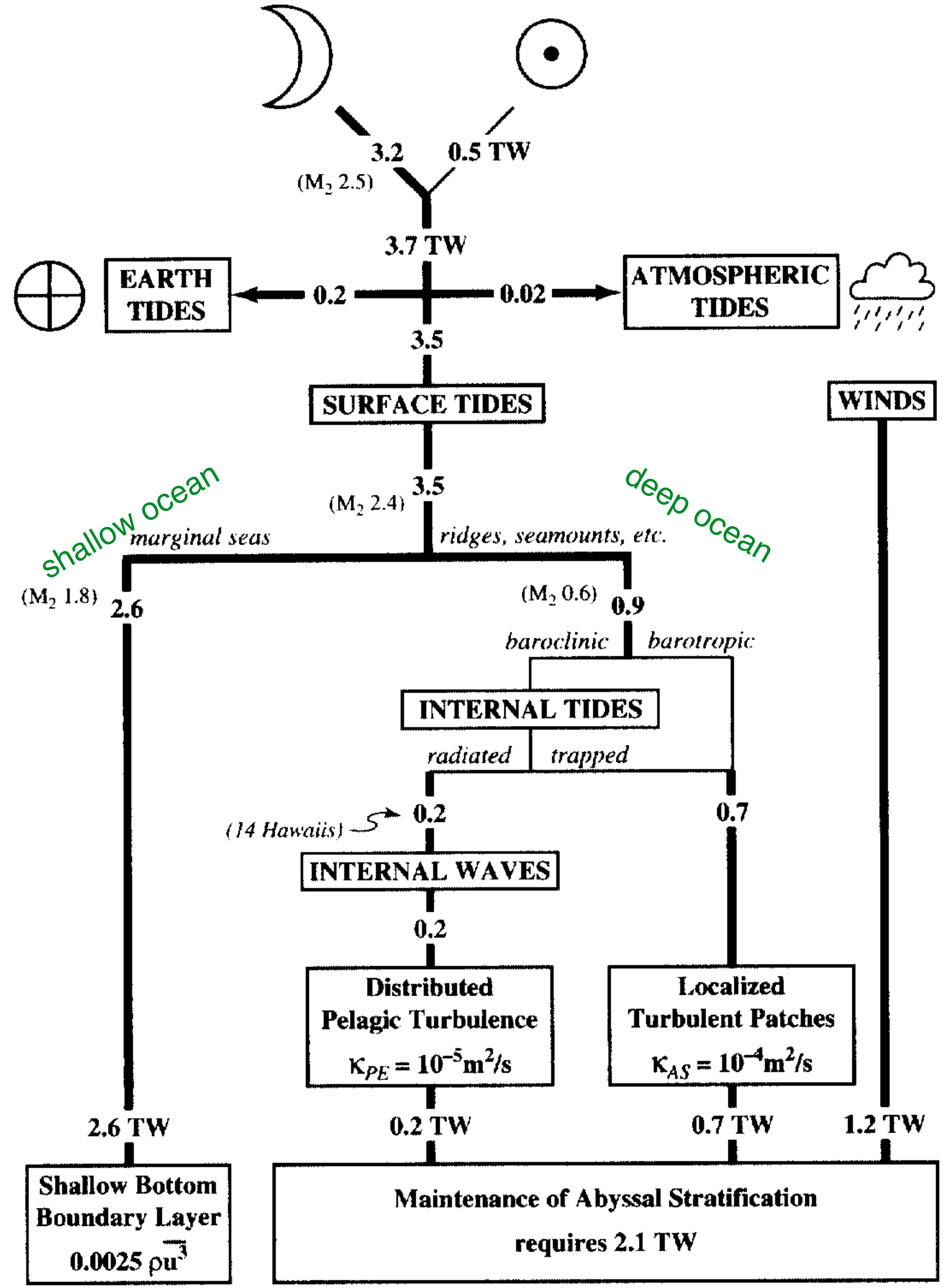
And what happens to the "lost spin energy"?

3.14TW becomes 3.7TW when the solar tides are included. But there is dissipation in the solid Earth, leaving 3.5TW for the ocean

There are two dissipative oceanic processes acting on the tide:

(1) the largest is turbulent drag in the BBL 70-75%;

(2) the more interesting is radiation of internal gravity waves 25-30%.



From Munk & Wunsch (1998) based on Egbert & Ray (2000, 2001)

Process 1: turbulent drag

Here is Taylor's 1919 estimate for the Irish Sea:

$$\varepsilon = \underbrace{c_D}_{0.002}\ \underbrace{\rho}_{1030\text{kg m}^{-3}}\ \underbrace{V^3}_{(1.14\text{m s}^{-1})^3}\ \underbrace{\langle|\cos^3\omega t|\rangle}_{4/3\pi}$$

$$= 1.3\text{W m}^{-2}$$

Integration over the area, gives 0.041TW in the Irish Sea.

Note: turbulent drag is most effective in shallow seas, where tidal velocities are large. It is negligible in the open ocean.

"The amount of dissipation found by the method just described is so much larger than any previous estimate of tidal friction that it is worth while to try and verify it, if possible, by some different method."

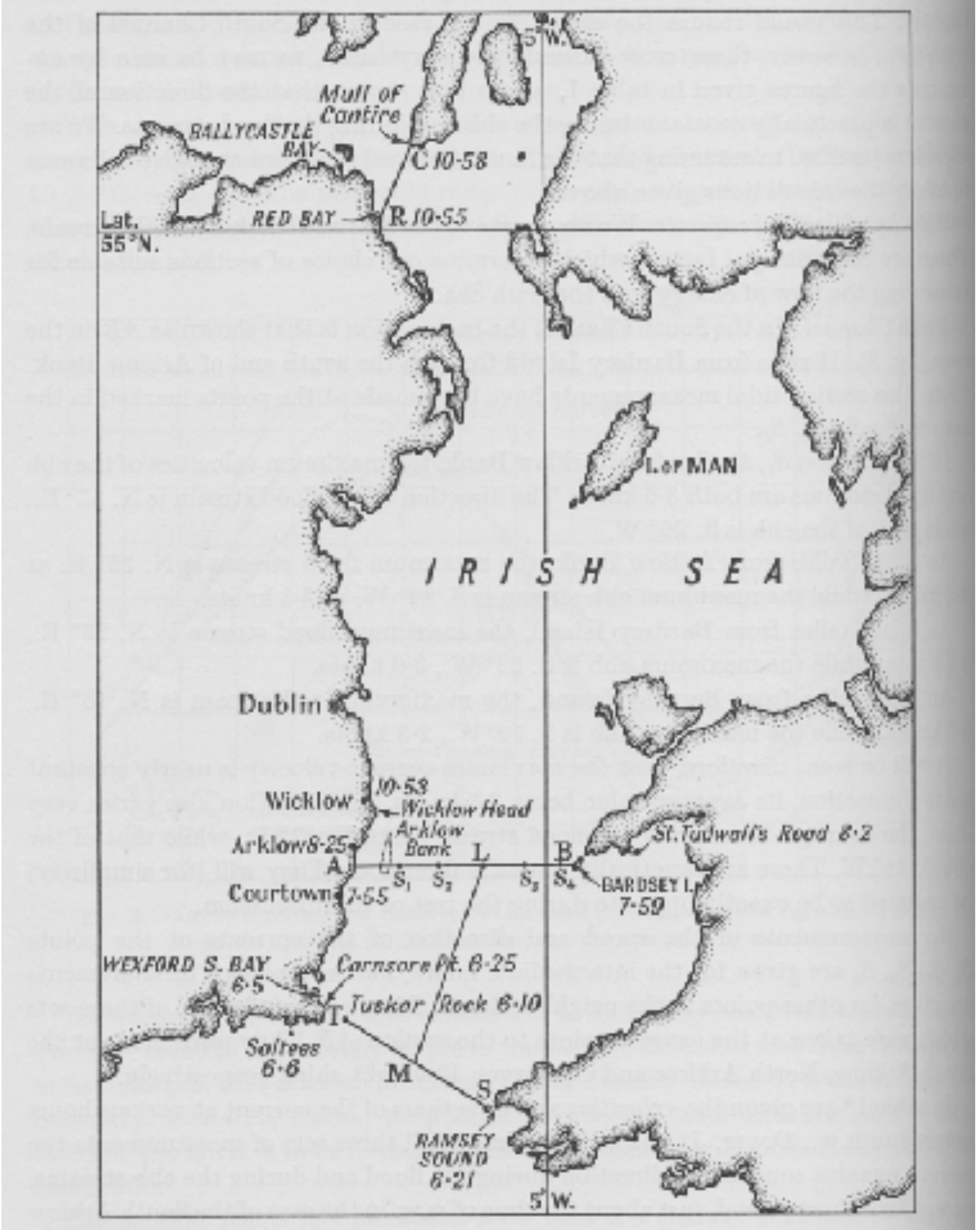
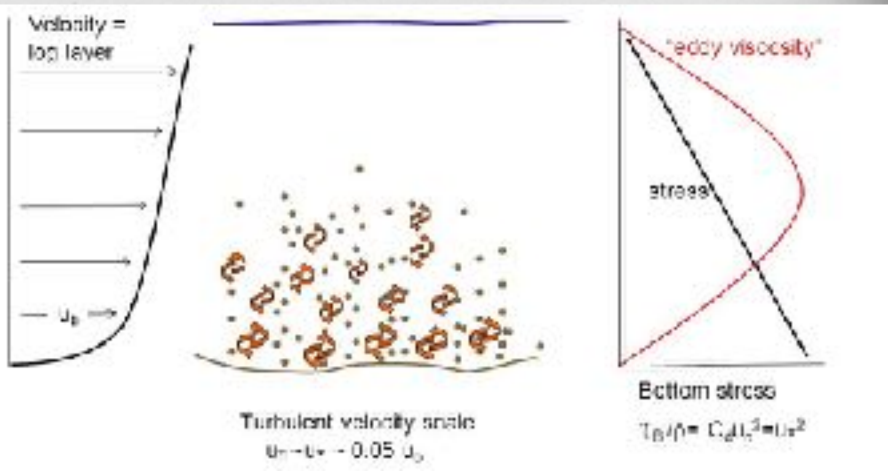


Fig. 3. Map of the Irish Sea. The figures are the times of H.W. at full and change of the moon. Thus Courtown 7·55 means that at Courtown it is H.W. at 7 hr. 55 min. at full and change of the moon.



The different method
(modern notation)

The LTEs (no self-attraction and loading).

$$\zeta_t + \boldsymbol{\nabla}\cdot(h\boldsymbol{u}) = 0$$

$$\boldsymbol{u}_t + \hat{\boldsymbol{z}}\times f\boldsymbol{u} = g\boldsymbol{\nabla}\left(\zeta^e - \zeta\right) + \boldsymbol{F}$$

Forcing is via the “equilibrium tide”

$$g\zeta^e = \frac{GM_{☾}R_{\oplus}^2}{s^3}P_2(\cos\psi) \qquad \cos\psi = \hat{\boldsymbol{r}}\cdot\hat{\boldsymbol{s}}$$

and $\boldsymbol{F} = \text{a dissipative force}$

The time-averaged energy equation is

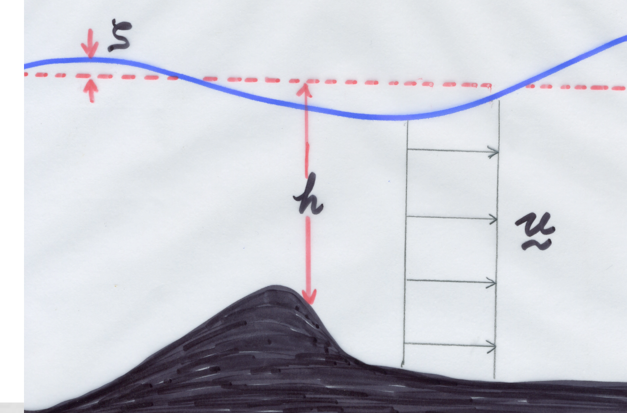
$$\boldsymbol{\nabla}\cdot\langle g\zeta h\boldsymbol{u}\rangle = \langle g\zeta_t\zeta^e\rangle + \langle h\boldsymbol{u}\cdot\boldsymbol{F}\rangle$$

Integrate over the Irish Sea:

$$\underbrace{\oint_{\partial R}\langle g\zeta h\boldsymbol{u}\rangle\cdot\boldsymbol{n}\,\mathrm{d}\ell}_{\text{boundary flux}} = \underbrace{\iint_R\langle g\zeta_t\zeta^e\rangle\,\mathrm{d}a}_{\text{local forcing}} + \underbrace{\iint_R\langle h\boldsymbol{u}\cdot\boldsymbol{F}\rangle\,\mathrm{d}a}_{\text{tidal dissipation}}$$

Infer the tidal dissipation by evaluating the other two terms using observations, or models, of the tide.

By this method the dissipation is 1.5W m^{-2}
The previous estimate was 1.3W m^{-2}



Flux through the North Channel is negligible.

Local forcing is small and negative.

Flux through the South Channel is the main source.

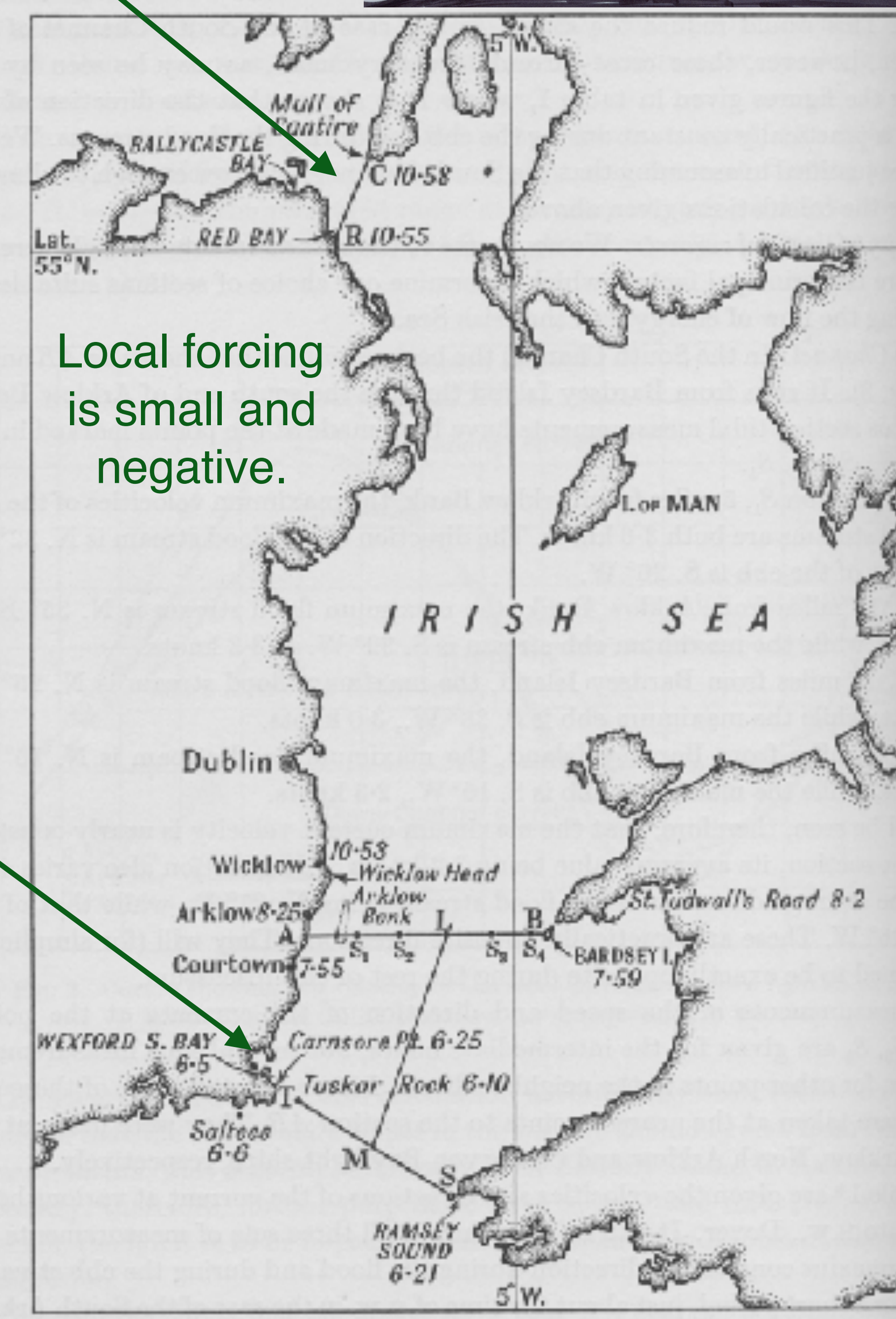


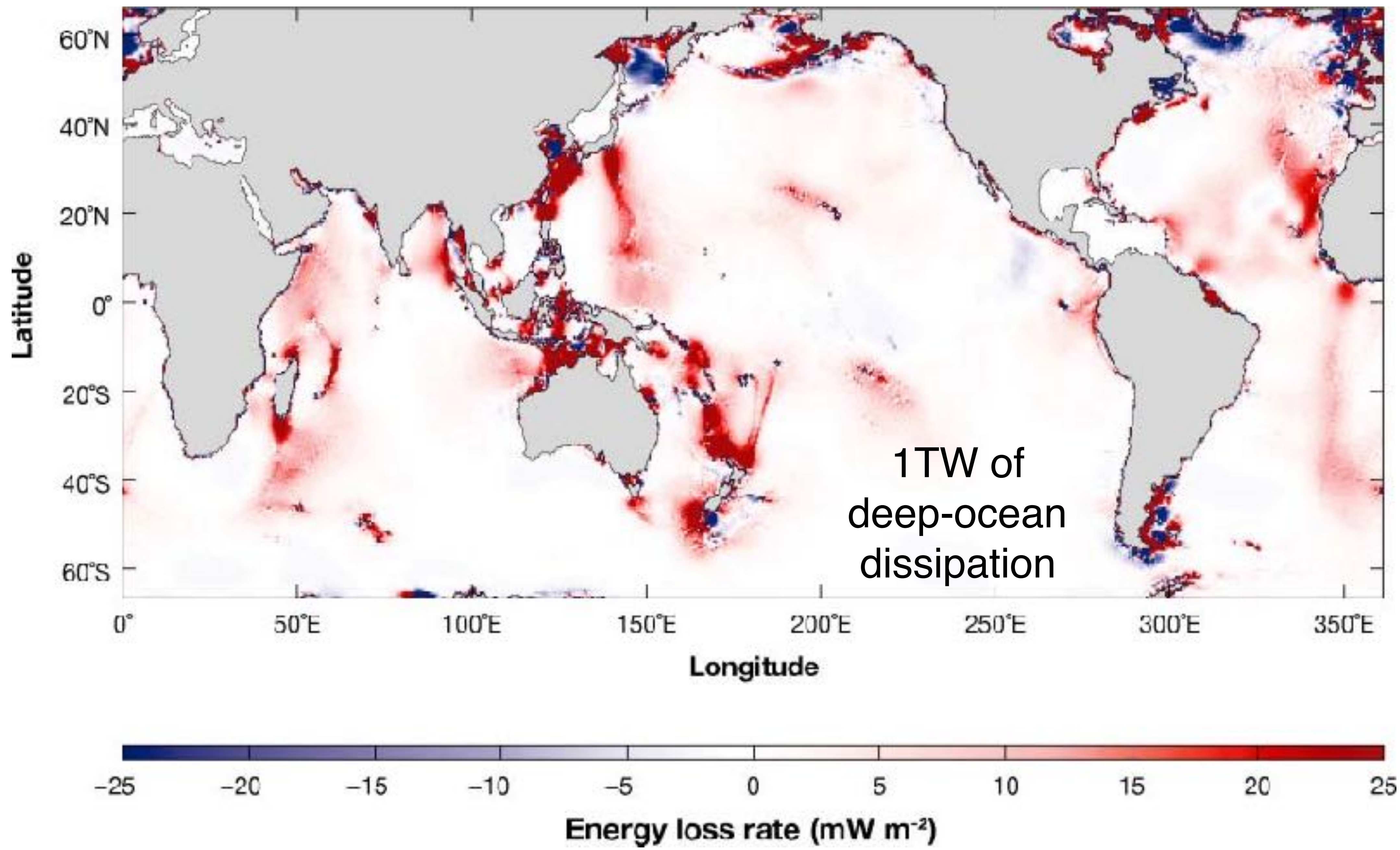
Fig. 3. Map of the Irish Sea. The figures are the times of H.W. at full and change of the mo
Thus Courtown 7·55 means that at Courtown it is H.W. at 7 hr. 55 min. at full and change
the moon.

“…the agreement between the two methods of estimating the energy dissipation due to tidal friction is quite remarkable.”

A modern version of Taylor’s “different method”

Empirically mapped tidal dissipation based on assimilative solution of the LTE, constrained by satellite measurements of ζ.

Egbert & Ray (2000)



Energy loss in the deep ocean is significantly more than can be attributed to turbulent drag on the bottom.

“All areas of consistently enhanced dissipation have significant bathymetric roughness, generally with elongated features such as ridges and island chains oriented perpendicular to tidal flows.”

Egbert & Ray (2000) shows that there is 1TW of deep-ocean tidal dissipation.

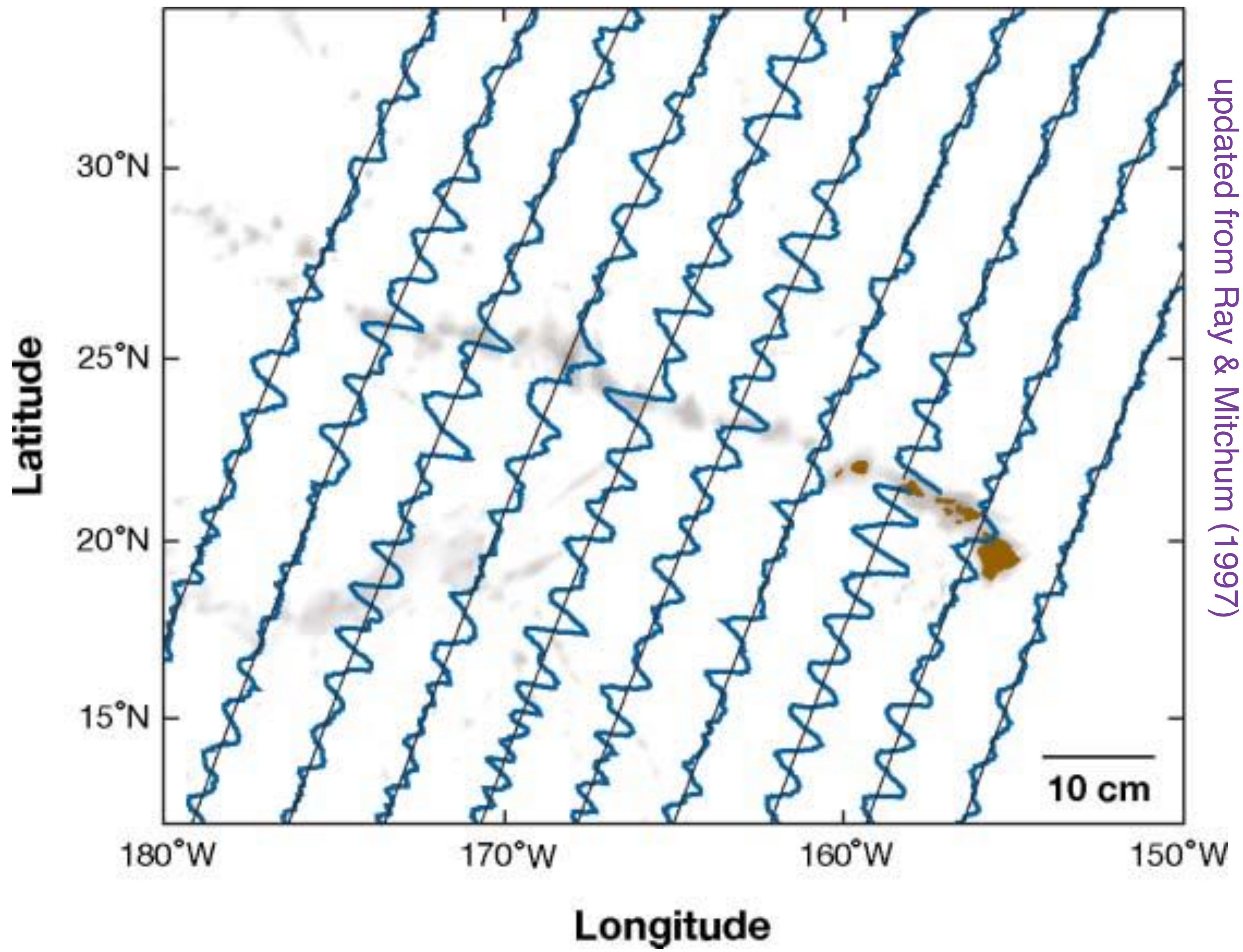
"Although our results do not explicitly constrain the mechanism of open-ocean tidal dissipation, the observed spatial pattern strongly suggests a significant role for scattering into internal tides, induced by tidal flow of the stratified ocean over rough topography."

Scattering into internal waves, or "tidal conversion", is an important source of ocean internal wave energy and of tidal friction. There are observations….

Process 2: radiation of IGWs

Satellite altimetry maps the elevation of the sea surface and shows semidiurnal tides propagating away from mid-ocean ridges. The wavelength corresponds to mode-one or mode-two internal tides.

“A significant fraction of the semidiurnal tide … is phase-locked to the astronomical potential and can modulate the amplitude of the surface tide by ~5cm … the internal tide is coherent over great distances, with waves propagating well over 1000km”



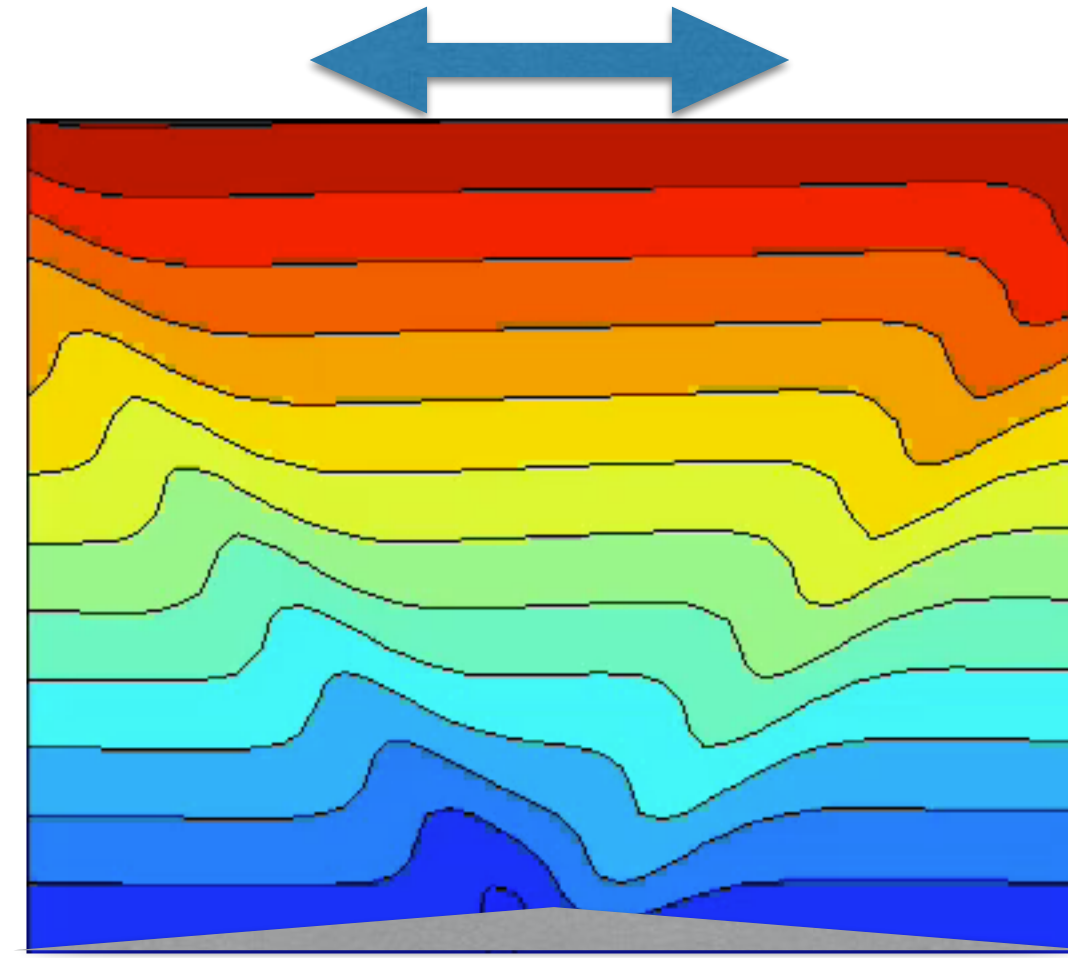
updated from Ray & Mitchum (1997)

The amplitude near Hawaii of the surface displacement at the M2 frequency (blue lines with the 10 cm scale shown), plotted normal to the TOPEX/Poseidon track (smooth gray lines) and high-passed with a cut-off scale of approximately 400 km. The large-scale pattern expected for the barotropic tide is alternately reinforced and reduced by motions with a much shorter wavelength.

Spectral analysis of the M_2 and S_2 wavetrains of track 125 (Figure 3) reveals two peaks at approximate wavelengths $(\lambda_1, \lambda_2) = (150, 85) \pm 10$ km for M_2, $(160, 80) \pm 20$ km for S_2. Assuming that wave propagation occurs parallel to the satellite groundtracks, these values give quite reasonable agreement with the theoretically expected wavelengths of the first

Tidal conversion — another lecture

$$U = U_0 \cos \omega_{M2} t$$



The barotropic deep-ocean tide sloshes stratified water over bottom bumps, resulting in IGW radiation.

These IGWs have tidal frequency and are known as the internal tide. Internal tides obey the IGW dispersion relation.

The linear problem can be solved analytically in some detail — particularly with the weak topography approximation (WTA).

$$\rho = \rho_0 \left(1 + g^{-1} \int_z^0 N^2(z')\, dz' - g^{-1} b \right),$$

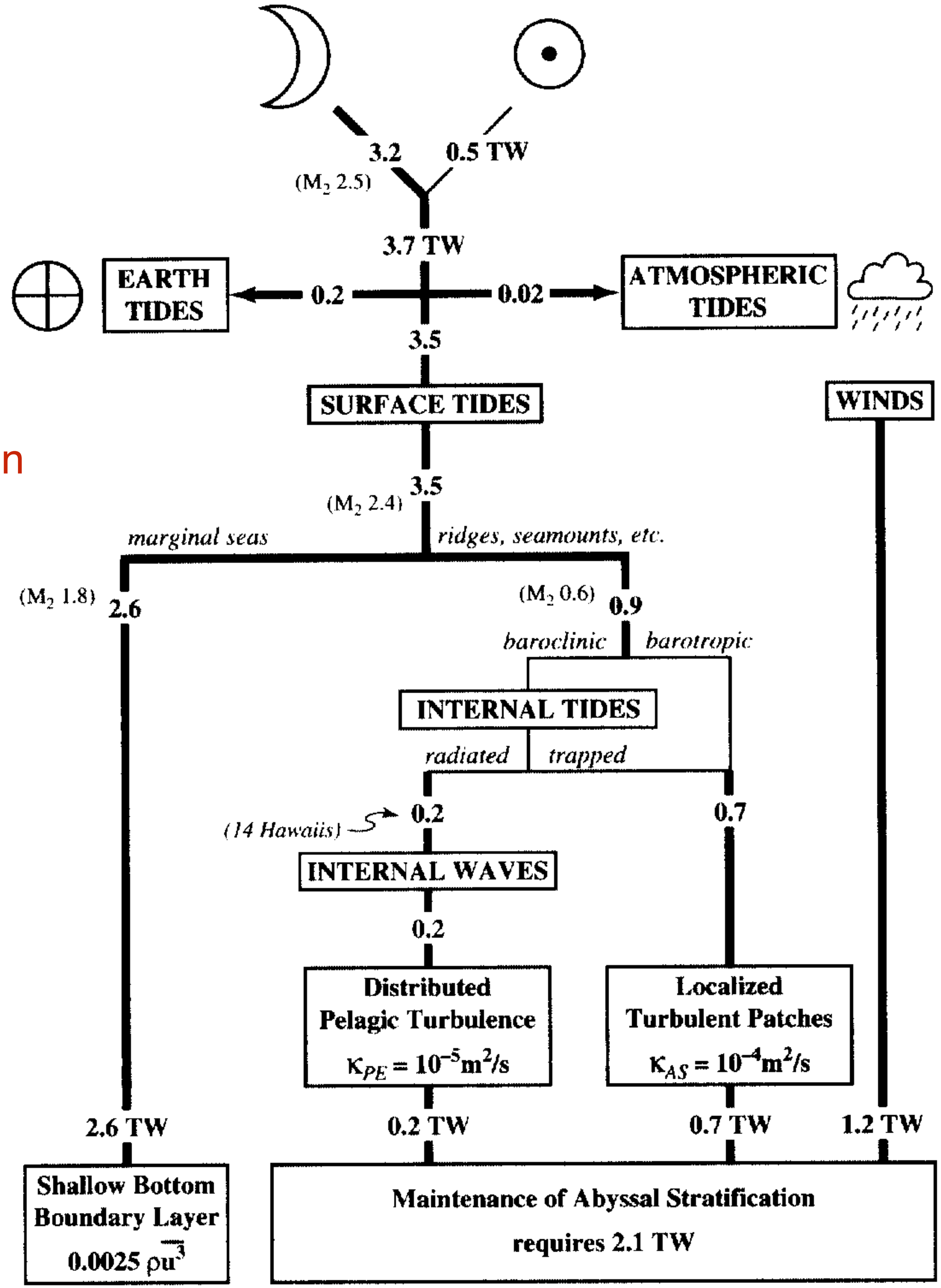
Note there is a useful small parameter (the "excursion parameter"). Both of these lengths are much smaller than the scale of the tidal flow.

$$\begin{aligned} \ell_{\mathrm{ex}} &= \frac{U_0}{\omega_{M2}} \\ &= \frac{0.05 \mathrm{m\ s}^{-1}}{1.41 \times 10^{-4} \mathrm{s}^{-1}} \\ &= 35.5 \mathrm{m} \ll \ell_{\mathrm{topo}} \end{aligned}$$

Stay tuned for

Maintenance of Abyssal Stratification

2.1TW



From Munk & Wunsch (1998)

Energetics of Tidal and wind mixing.


THE END?