Turbulent Flows in Climate Dynamics

Les Houches, Chamonix Valley
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Climate Change & Climate Variability: A Unified Framework

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Please visit these sites for more info.

https://dept.atmos.ucla.edu/tcd/, http://www.environnement.ens.fr/
and https://www.researchgate.net/profile/Michael_Ghil

Turbulence through the ages

"I am an old man now, and when I die and go to Heaven there are two matters on which I hope for enlightenment. One is quantum electrodynamics, and the other is the turbulent motion of fluids. And about the former I am really rather optimistic."

— Sir Horace Lamb, 1932

"Lamb was correct on two scores. All who knew him agreed that it was Heaven that he would go to, and he was right to be more optimistic about quantum electrodynamics than turbulence."

— Sydney Goldstein, 1969

Goldstein, S., 1969: Fluid mechanics in the first half of this century. *Annu. Rev. Fluid Mech.*, **1**, 1–29.

Sydney Goldstein

Outline – Dynamical systems & climate sensitivity

- The IPCC process: results and uncertainties
- Natural climate variability as a source of uncertainties
  - sensitivity to initial data ➔ error growth
  - sensitivity to model formulation ➔ see below!
- Uncertainties and how to fix them
  - structural stability and other kinds of robustness
  - non-autonomous and random dynamical systems (NDDS & RDS)
- Illustrative examples
  - the Lorenz convection model
  - an El Niño–Southern Oscillation (ENSO) model
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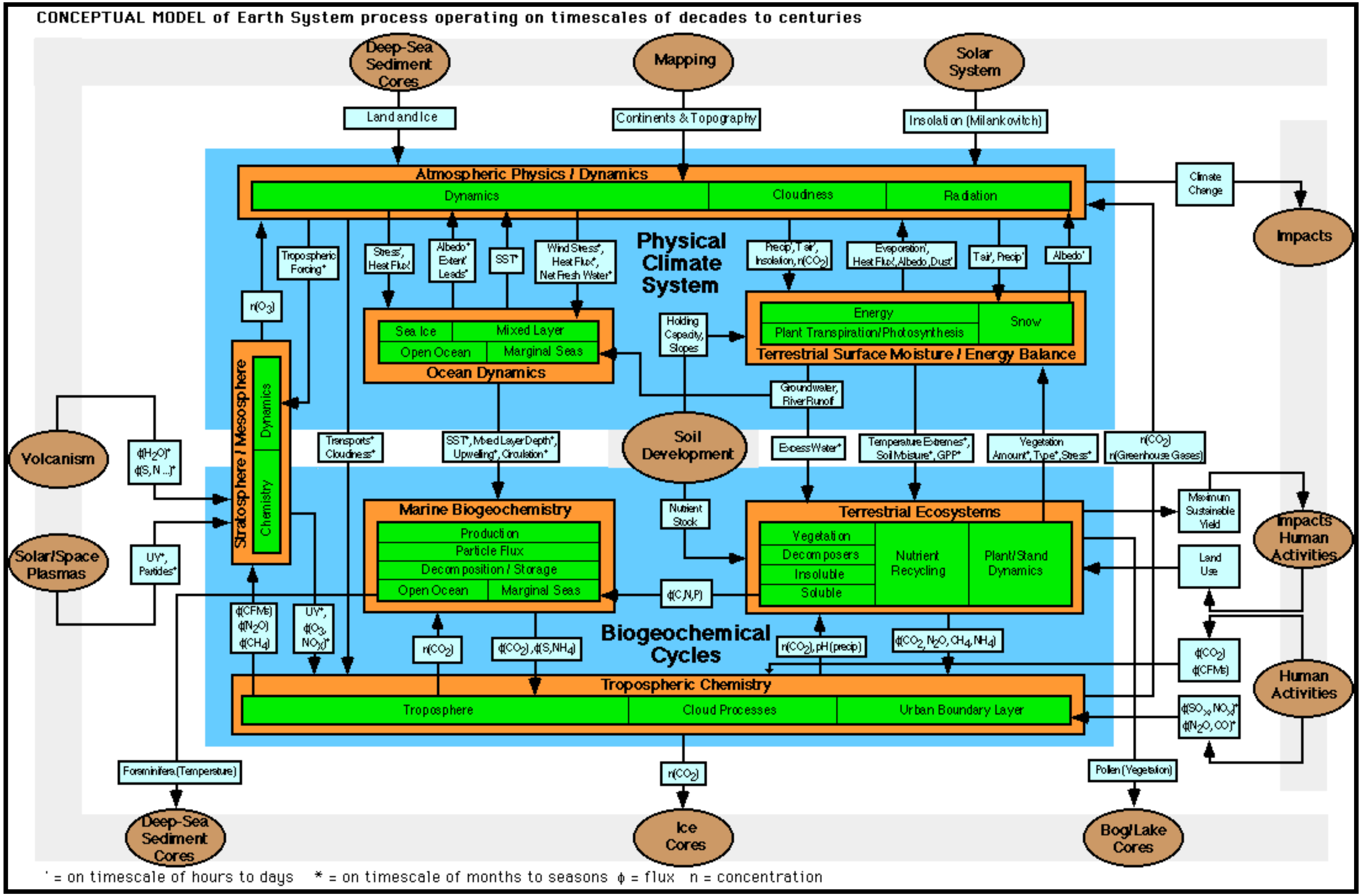
  - selected bibliography

Motivation

- The ***climate system*** is highly ***nonlinear and*** quite ***complex***.
- The system's ***major components*** — the atmosphere, oceans, ice sheets — ***evolve*** on many time and space scales.
- Its ***predictive understanding*** has to rely on the system's physical, chemical and biological modeling, but also on the thorough analysis of the models thus obtained
  - ***deterministic vs. stochastic***
  - ***closed (autonomous) vs. open (non-autonomous)***
- The ***hierarchical modeling*** approach allows one to give proper weight to the understanding provided by the models *vs*. their realism: back-and-forth between ***"toy"*** (conceptual) and ***detailed*** ("realistic") ***models***, and between ***models*** and ***data***.
- How do we disentangle ***natural (intrinsic, endogenous) variability*** from the ***anthropogenic (extrinsic, exogenous) forcing?***

F. Bretherton's "horrendogram" of Earth System Science



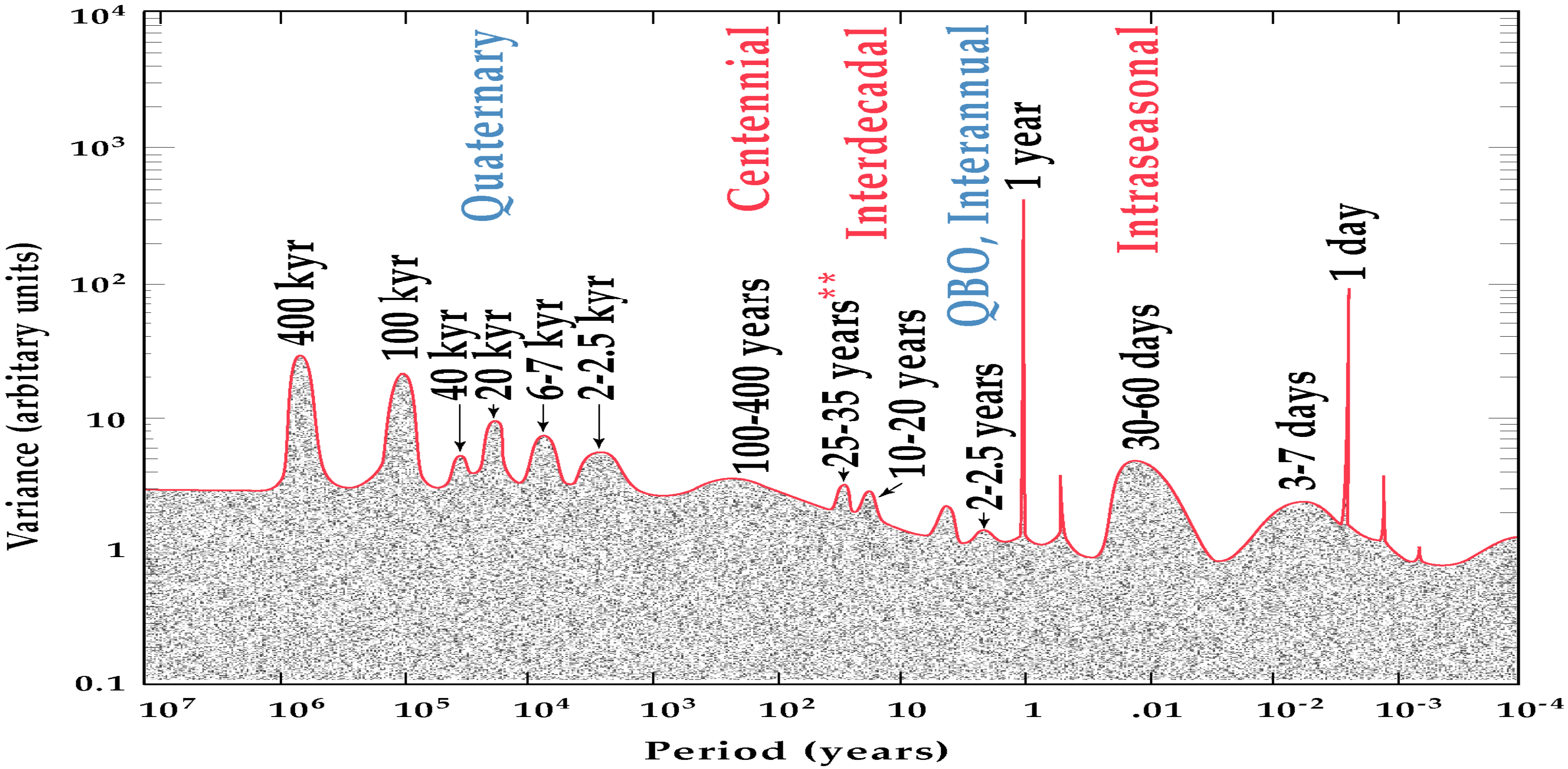
Earth System Science Overview, *NASA Advisory Council*, 1986

Composite spectrum of climate variability

Standard treatement of frequency bands:

1. High frequencies – white noise (or "colored")

2. Low frequencies – slow evolution of parameters



From **Ghil (2001, *EGEC*)**, after **Mitchell* (1976)**

*** "No known source of deterministic internal variability"**

**** 27 years – Brier (1968, *Rev. Geophys.*)**

Climate and Its Sensitivity

Let's say CO_2 doubles:

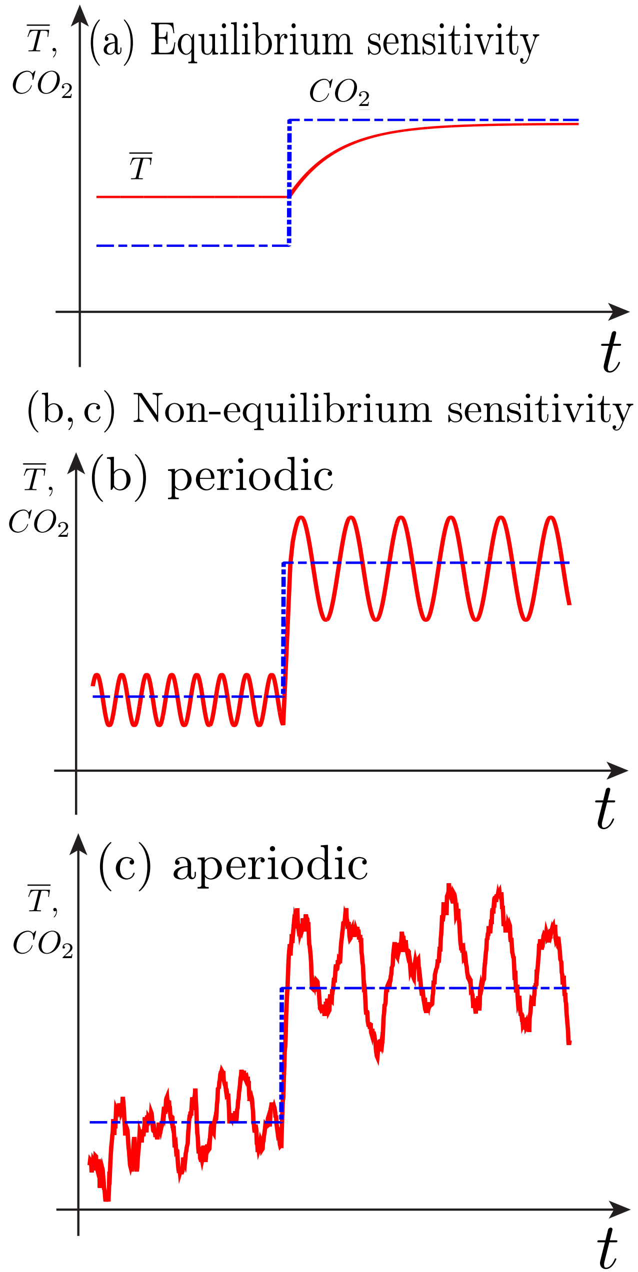
How will "climate" change?

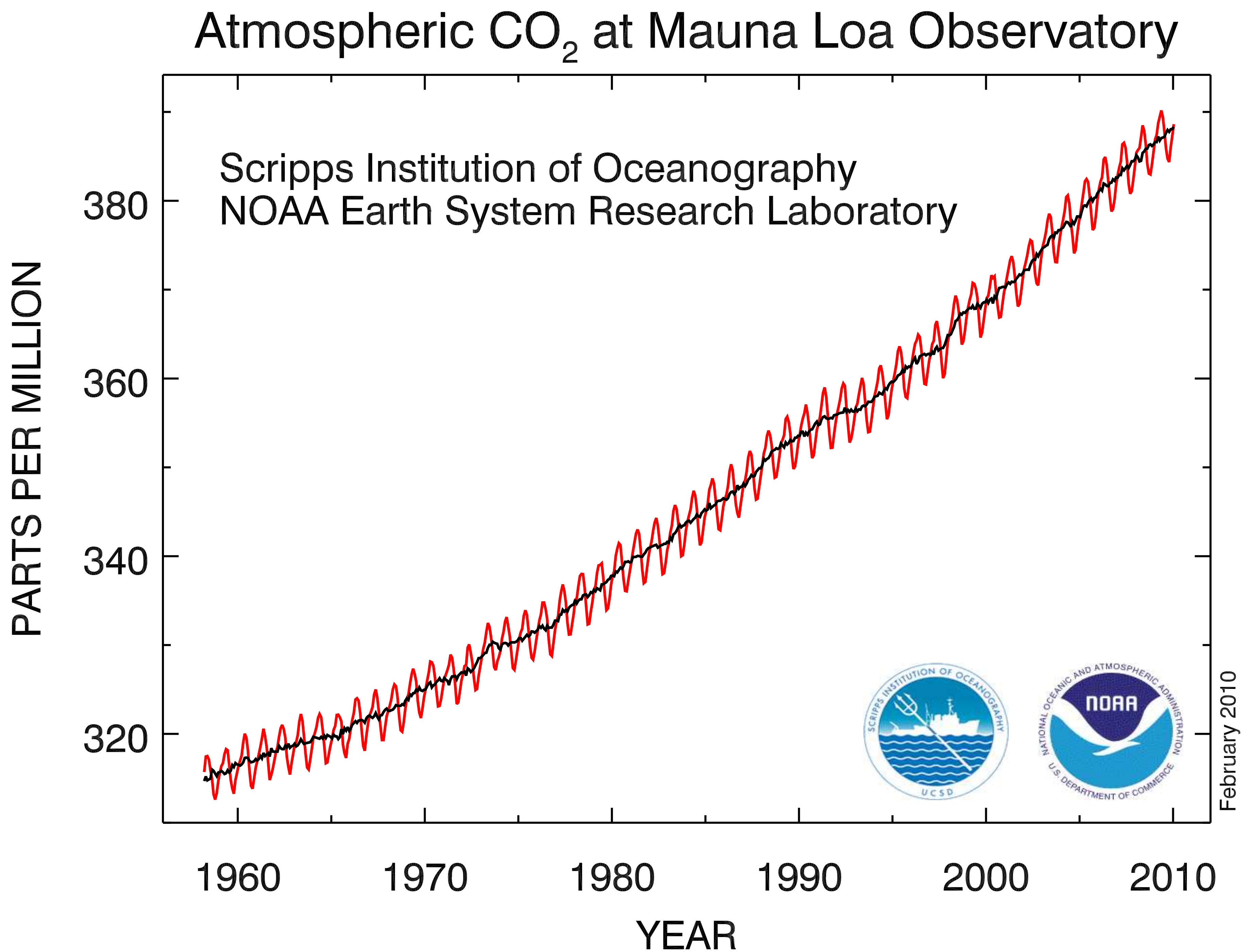
1. Climate is in stable equilibrium (fixed point); if so, mean temperature will just shift gradually to its new equilibrium value.

2. Climate is purely periodic; if so, mean temperature will (maybe) shift gradually to its new equilibrium value. But how will the period, amplitude and phase of the limit cycle change?

3. And how about some "real stuff" now: chaotic + random?

Ghil (in *Encycl. Global Environmental Change*, 2002)

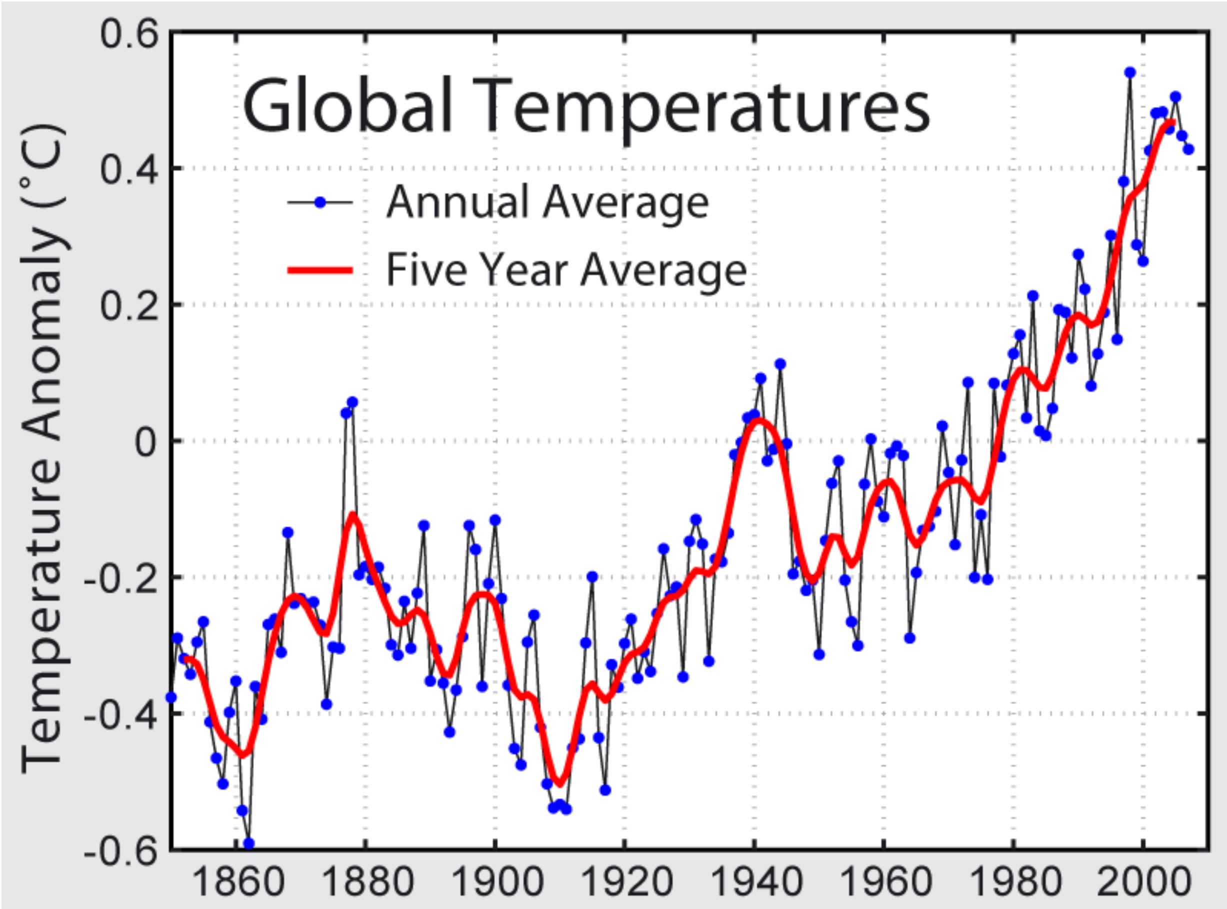



Atmospheric CO_2 at Mauna Loa Observatory
Scripps Institution of Oceanography
NOAA Earth System Research Laboratory
PARTS PER MILLION
320
340
360
380
1960
1970
1980
1990
2000
2010
YEAR
February 2010

Temperatures and GHGs

Greenhouse gases (GHGs) go up,
temperatures go up:

It's gotta do with us, at least a bit,
doesn't it?



Wikicommons, from
Hansen *et al*. (*PNAS*, 2006);
see also http://data.giss.nasa.gov/gistemp/graphs/

Unfortunately, things aren't all that easy!

What to do?

Try to achieve better interpretation of, and agreement between, models …

Ghil, M., 2002: Natural climate variability, in *Encyclopedia of Global Environmental Change*, T. Munn (Ed.), Vol. 1, Wiley

Natural variability introduces additional complexity into the anthropogenic climate change problem

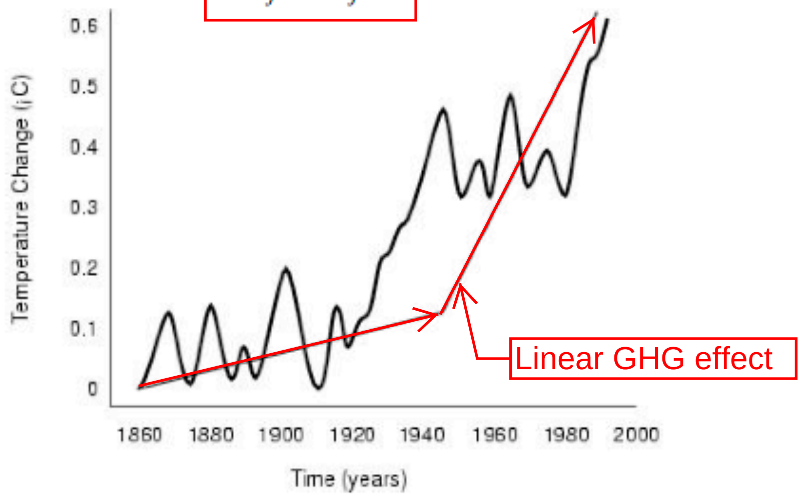
The most common interpretation of observations and GCM simulations of climate change is still in terms of a scalar, linear Ordinary Differential Equation (ODE)

$$\boxed{c\frac{dT}{dt} = -kT + Q}$$

$$\boxed{\begin{aligned} k &= \sum k_i \\ Q &= \sum Q_j \\ Q_j &= Q_j(t) \end{aligned}}$$

– feedbacks (+ve and -ve)

– sources & sinks



Linear response to CO_2 vs. observed change in T

Hence, we need to consider instead a system of nonlinear Partial Differential Equations (PDEs), with parameters and multiplicative, as well as additive forcing (deterministic + stochastic)

$$\boxed{\frac{d\boldsymbol{X}}{dt} = \boldsymbol{N}(\boldsymbol{X}, t, \mu, \beta)}$$

Global warming and its socio-economic impacts

Temperatures rise:

- What about impacts?
- How to adapt?

The answer, my friend, is blowing in the wind, *i.e.*, it depends on the accuracy and reliability of the forecast …

Source : IPCC (2007), *AR4, WGI, SPM*

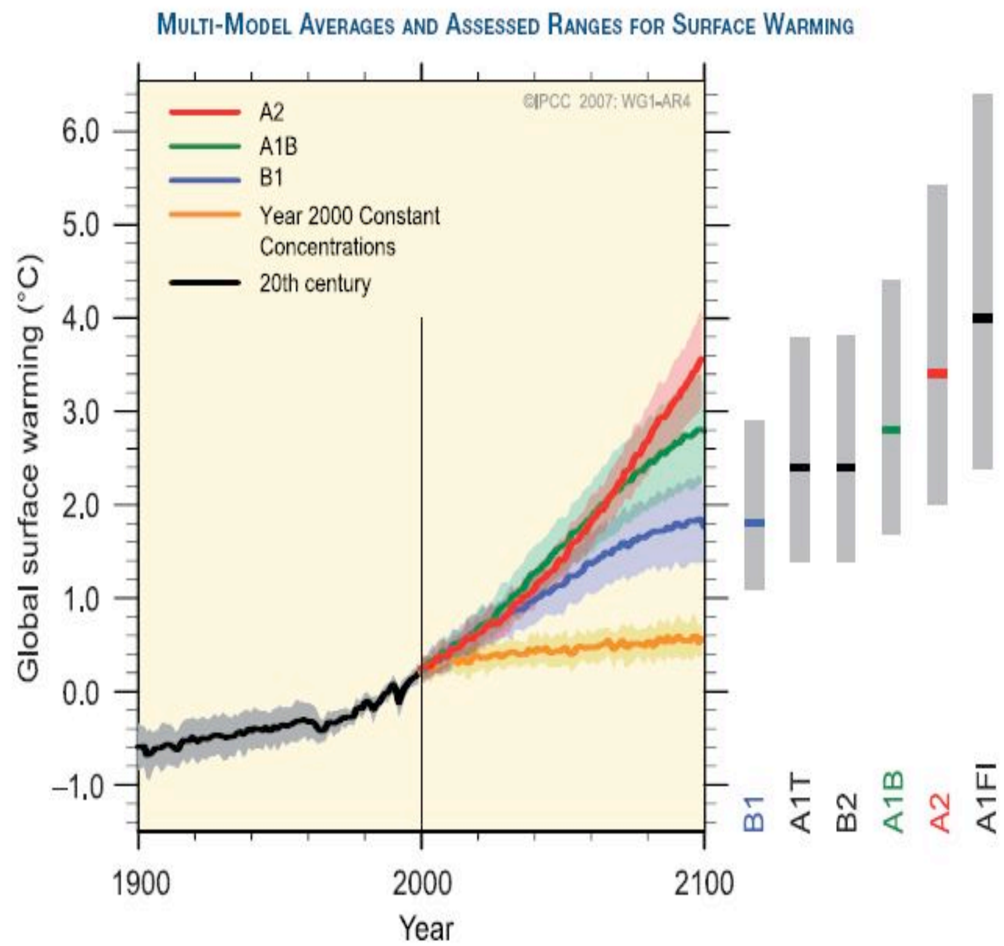


Figure SPM.5. *Solid lines are multi-model global averages of surface warming (relative to 1980–1999) for the scenarios A2, A1B and B1, shown as continuations of the 20th century simulations. Shading denotes the ±1 standard deviation range of individual model annual averages. The orange line is for the experiment where concentrations were held constant at year 2000 values. The grey bars at right indicate the best estimate (solid line within each bar) and the* **likely** *range assessed for the six SRES marker scenarios. The assessment of the best estimate and* **likely** *ranges in the grey bars includes the AOGCMs in the left part of the figure, as well as results from a hierarchy of independent models and observational constraints. {Figures 10.4 and 10.29}*

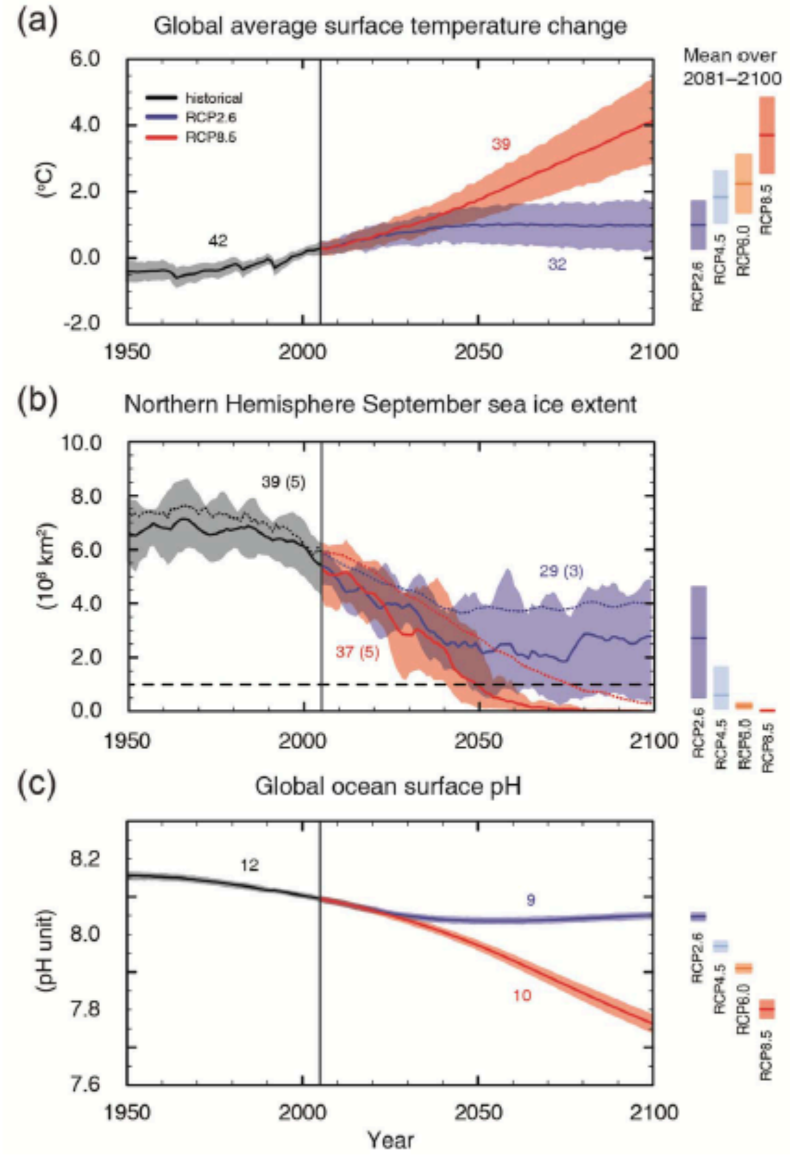
Global warming and its socio-economic impacts– II

Temperatures rise:

- What about impacts?
- How to adapt?

AR5 vs. AR4

A certain air of *déjà vu*:
GHG "scenarios" have been replaced by "representative concentration pathways" (RCPs), more dire predictions, but the uncertainties remain.



Source : IPCC (2013), AR5, WGI, SPM

Outline – Dynamical systems & climate sensitivity

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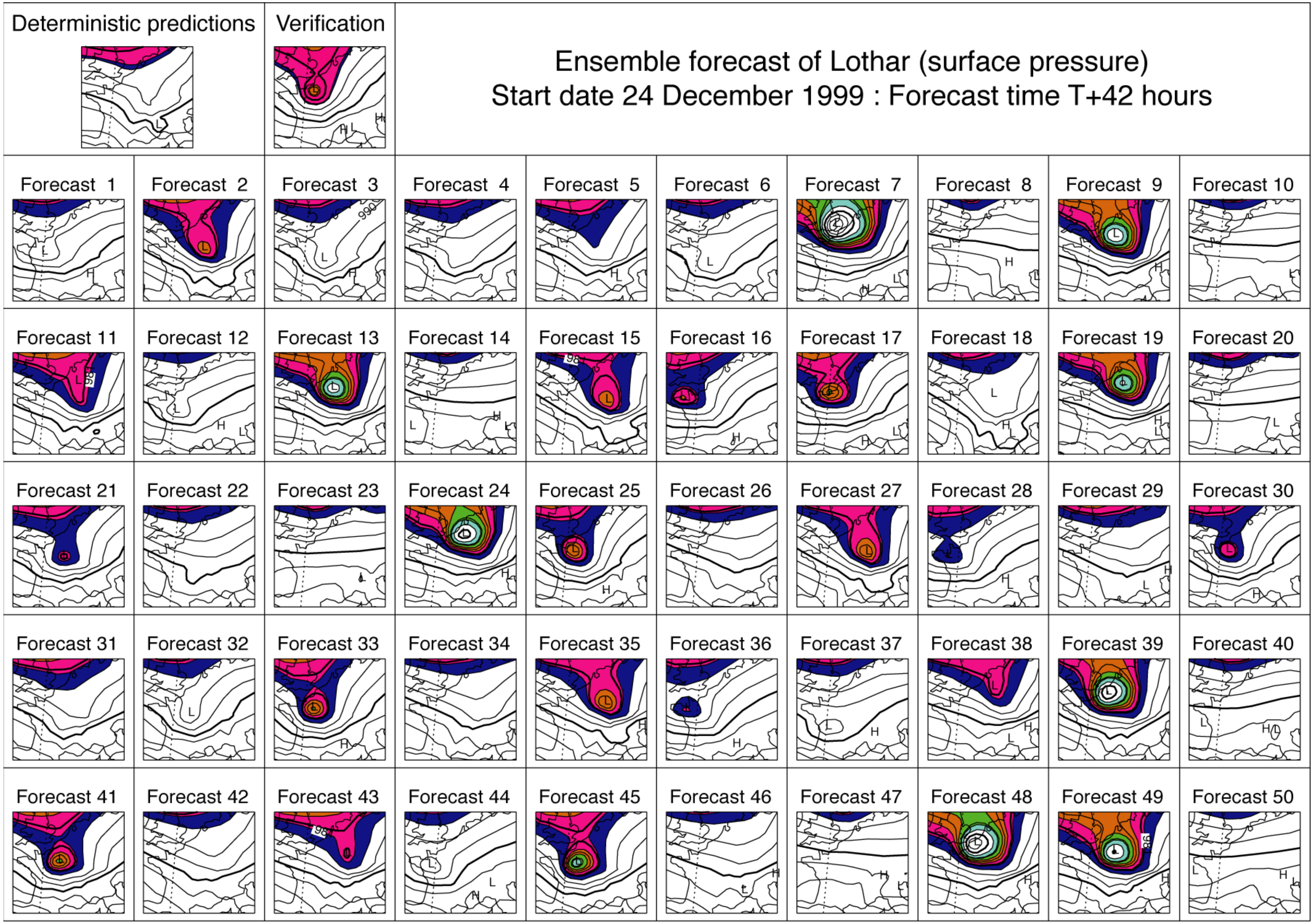


Courtesy Tim Palmer, 2009

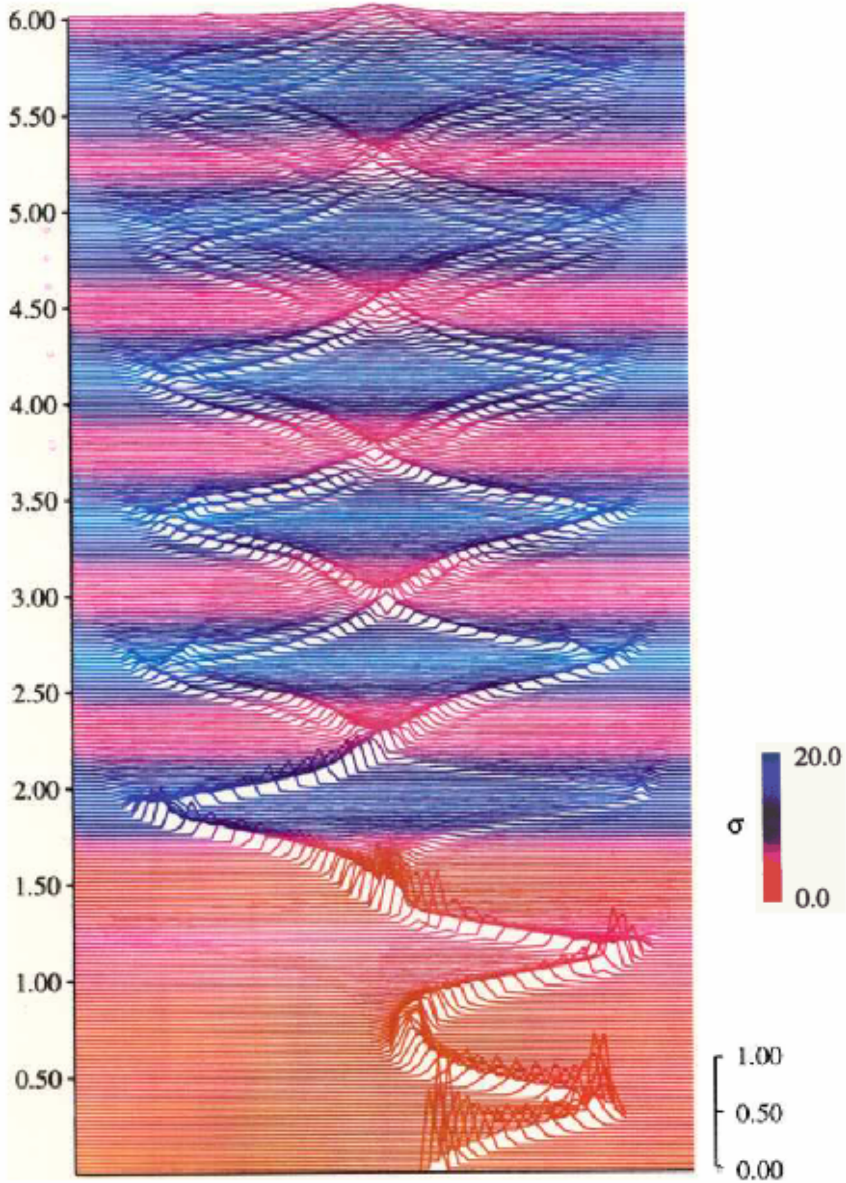
Exponential divergence vs. “coarse graining”

The classical view of dynamical systems theory is:

positive Lyapunov exponent ➔ trajectories diverge exponentially

But the presence of multiple regimes implies a much more structured behavior in phase space

Still, the probability distribution function (pdf), when calculated forward in time, is pretty smeared out



L. A. Smith (*Encycl. Atmos. Sci.*, 2003)

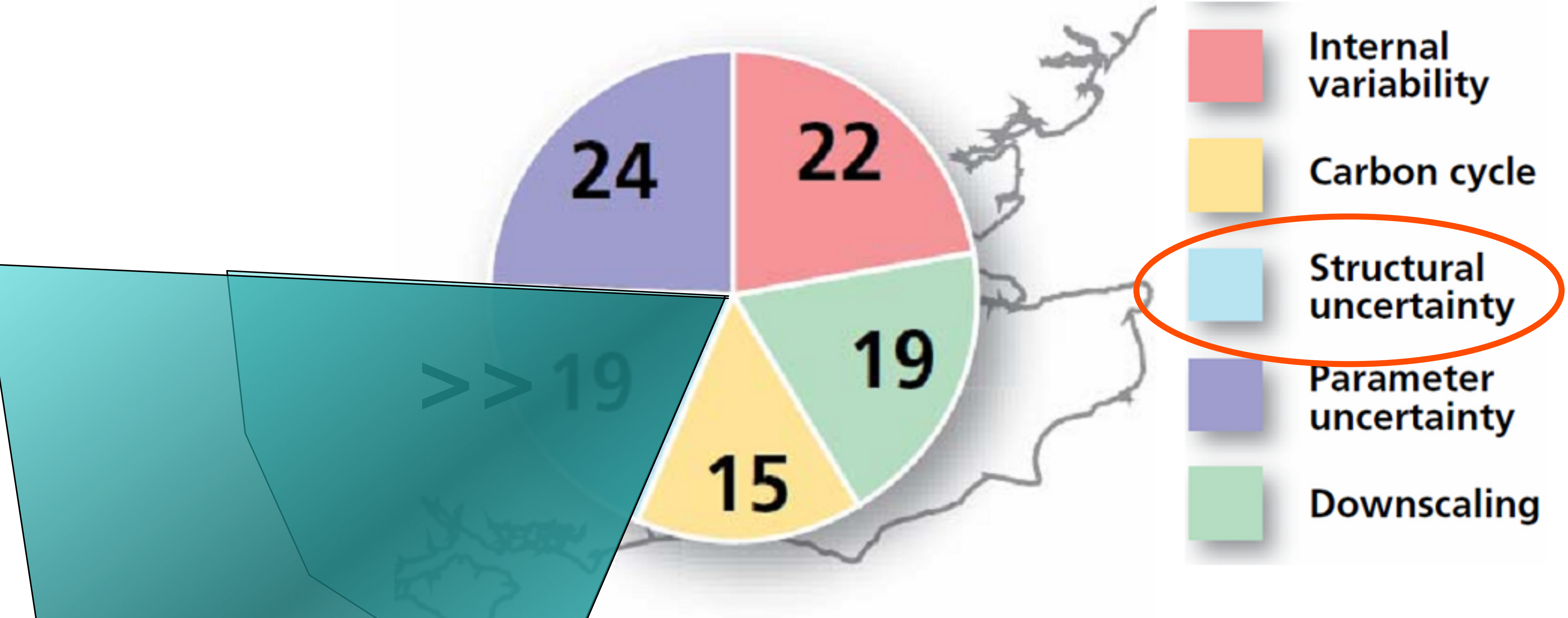
So what's it gonna be like, by 2100?

Table SPM.2. *Recent trends, assessment of human influence on the trend and projections for extreme weather events for which there is an observed late-20th century trend.* {Tables 3.7, 3.8, 9.4; Sections 3.8, 5.5, 9.7, 11.2–11.9}

Phenomenon[a] and direction of trend	Likelihood that trend occurred in late 20th century (typically post 1960)	Likelihood of a human contribution to observed trend[b]	Likelihood of future trends based on projections for 21st century using SRES scenarios
Warmer and fewer cold days and nights over most land areas	*Very likely*[c]	*Likely*[d]	*Virtually certain*[d]
Warmer and more frequent hot days and nights over most land areas	*Very likely*[e]	*Likely (nights)*[d]	*Virtually certain*[d]
Warm spells/heat waves. Frequency increases over most land areas	*Likely*	*More likely than not*[f]	*Very likely*
Heavy precipitation events. Frequency (or proportion of total rainfall from heavy falls) increases over most areas	*Likely*	*More likely than not*[f]	*Very likely*
Area affected by droughts increases	*Likely* in many regions since 1970s	*More likely than not*	*Likely*
Intense tropical cyclone activity increases	*Likely* in some regions since 1970	*More likely than not*[f]	*Likely*
Increased incidence of extreme high sea level (excludes tsunamis)[g]	*Likely*	*More likely than not*[f,h]	*Likely*[i]

How important are different sources of uncertainty?

- Varies, but typically no single source dominates.



Uncertainties in winter precipitation changes for the 2080s relative to 1961-90, at a 25km box in SE England

Source: Met Office

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Non-autonomous Dynamical Systems

A linear, dissipative, forced example: *forward vs. pullback attraction*

Consider the scalar, linear ordinary differential equation (ODE)

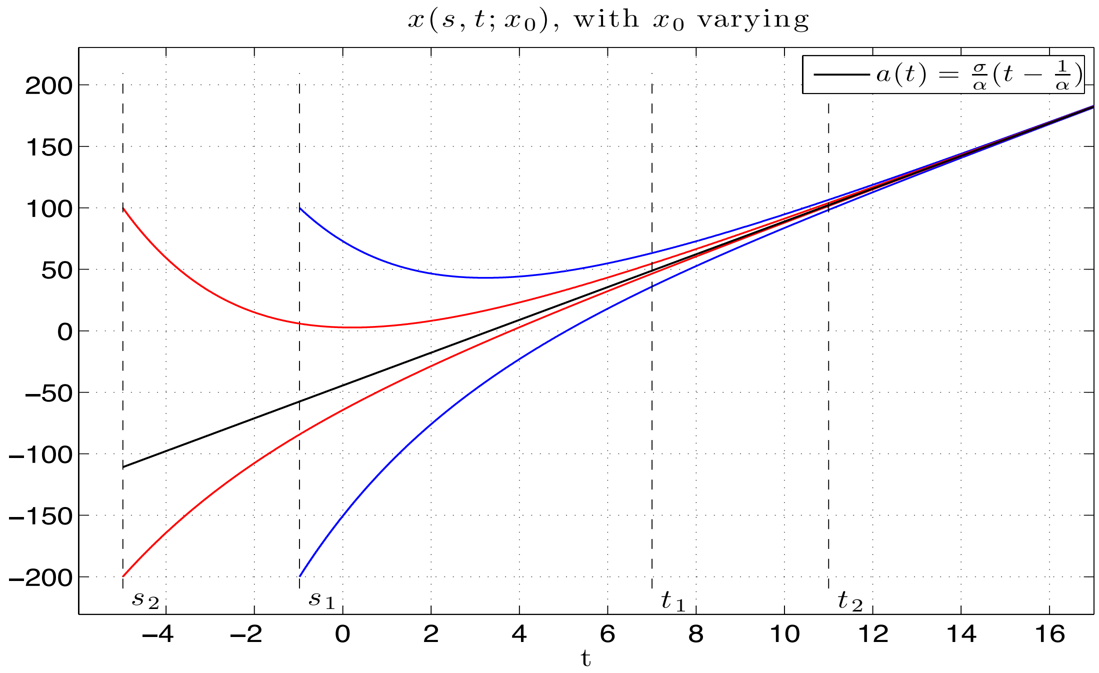
$$\dot{x} = -\alpha x + \sigma t\,, \quad \alpha > 0\,,\ \sigma > 0\,.$$

The autonomous part of this ODE, $\dot{x} = -\alpha x$, is dissipative and all solutions $x(t; x_0) = x(t; x(0) = x_0)$ converge to 0 as $t \to +\infty$.

What about the non-autonomous, forced ODE? As the energy being put into the system by the forcing is dissipated, we expect things to change in time. In fact, if we “pull back” far enough, replace $x(t; x_0)$ by $x(s, t; x_0) = x(s, t; x(s) = x_0)$,

and let $s \to -\infty$, we get th pullback attractor $a = a(t)$ in the figure,

$$a(t) = \frac{\sigma}{\alpha}\left(t - \frac{1}{\alpha}\right).$$



This theory is the counterpart for randomly forced dynamical systems (RDS) of the *geometric theory* of ordinary differential equations (ODEs). It allows one to treat stochastic differential equations (SDEs) — and more general systems driven by noise — as flows in (phase space)$\times$(probability space).

SDE$\sim$ODE, RDS$\sim$DDS, L. Arnold (1998)$\sim$V.I. Arnol'd (1983).

Setting:

(i) A phase space X. **Example**: $\mathbb{R}^n$.

(ii) A probability space $(\Omega, \mathcal{F}, \mathbb{P})$. **Example**: The Wiener space $\Omega = C_0(\mathbb{R}; \mathbb{R}^n)$ with Wiener measure $\mathbb{P}$.

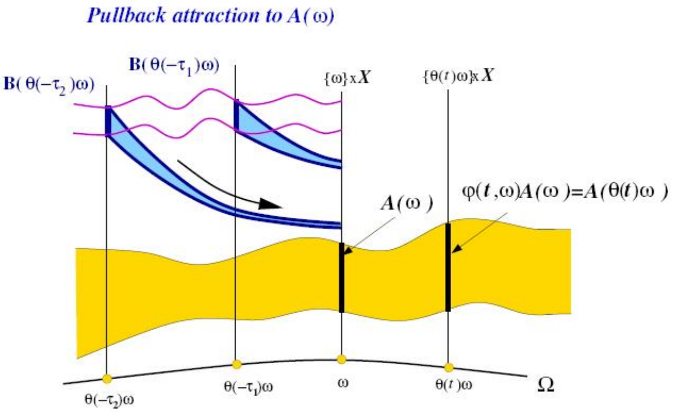
(iii) A model of the noise $\theta(t) : \Omega \to \Omega$ that preserves the measure $\mathbb{P}$, i.e. $\theta(t)\mathbb{P} = \mathbb{P}$; θ is called the driving system.
Example: $W(t, \theta(s)\omega) = W(t + s, \omega) - W(s, \omega)$;
it starts the noise at s instead of $t = 0$.

(iv) A mapping $\varphi : \mathbb{R} \times \Omega \times X \to X$ with the cocycle property.
Example: The solution operator of an SDE.

A random attractor $\mathcal{A}(\omega)$ is both *invariant* and "pullback" *attracting*:

(a) **Invariant**: $\varphi(t,\omega)\mathcal{A}(\omega) = \mathcal{A}(\theta(t)\omega)$.

(b) **Attracting**: $\forall B \subset X$, $\lim_{t\to\infty} \mathrm{dist}(\varphi(t, \theta(-t)\omega)B, \mathcal{A}(\omega)) = 0$ a.s.



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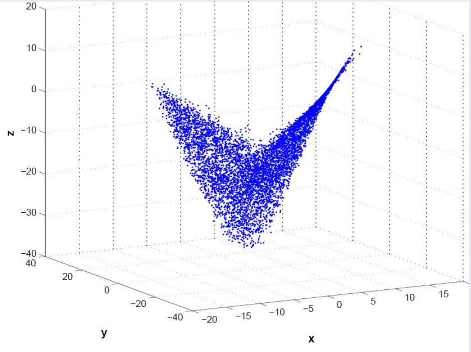
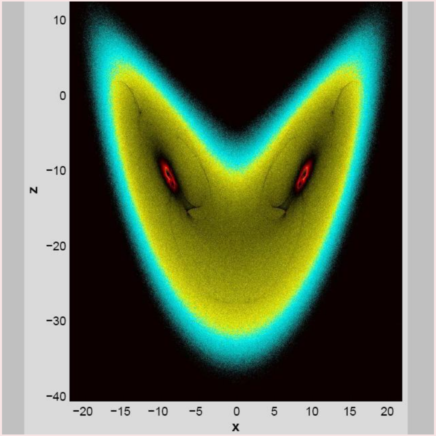
Random attractor of the stochastic Lorenz system

Snapshot of the random attractor (RA)



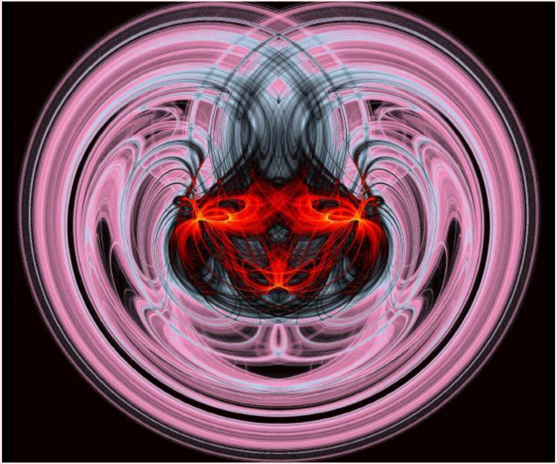
- A snapshot of the RA, $\mathcal{A}(\omega)$, computed at a fixed time t and for the same realization ω; it is made up of points transported by the stochastic flow, from the remote past $t - T$, $T >> 1$.
- We use multiplicative noise in the deterministic Lorenz model, with the classical parameter values $b = 8/3$, $\sigma = 10$, and $r = 28$.
- Even computed pathwise, this object supports meaningful statistics.

Sample measures supported by the R.A.



- We compute the probability measure on the R.A. at some fixed time t, and for a fixed realization ω. We show a "projection", $\int \mu_\omega(x, y, z)\mathrm{d}y$, with multiplicative noise: $\mathrm{d}x_i$=Lorenz$(x_1, x_2, x_3)\mathrm{d}t + \alpha\ x_i \mathrm{d}W_t; i \in \{1, 2, 3\}$.
- 10 million of initial points have been used for this picture!

Sample measure supported by the R.A.



- Still 1 Billion I.D., and $\alpha = 0.5$. Another one?

Sample measure supported by the R.A.

Sample measures evolve with time.

- Recall that these sample measures are the frozen statistics at a time *t* for a realization ω.
- How do these frozen statistics evolve with time?
- Action!

A day in the life of the Lorenz (1963) model's random attractor, or LORA for short;
see SI in Chekroun, Simonnet & Ghil (2011, *Physica D*)

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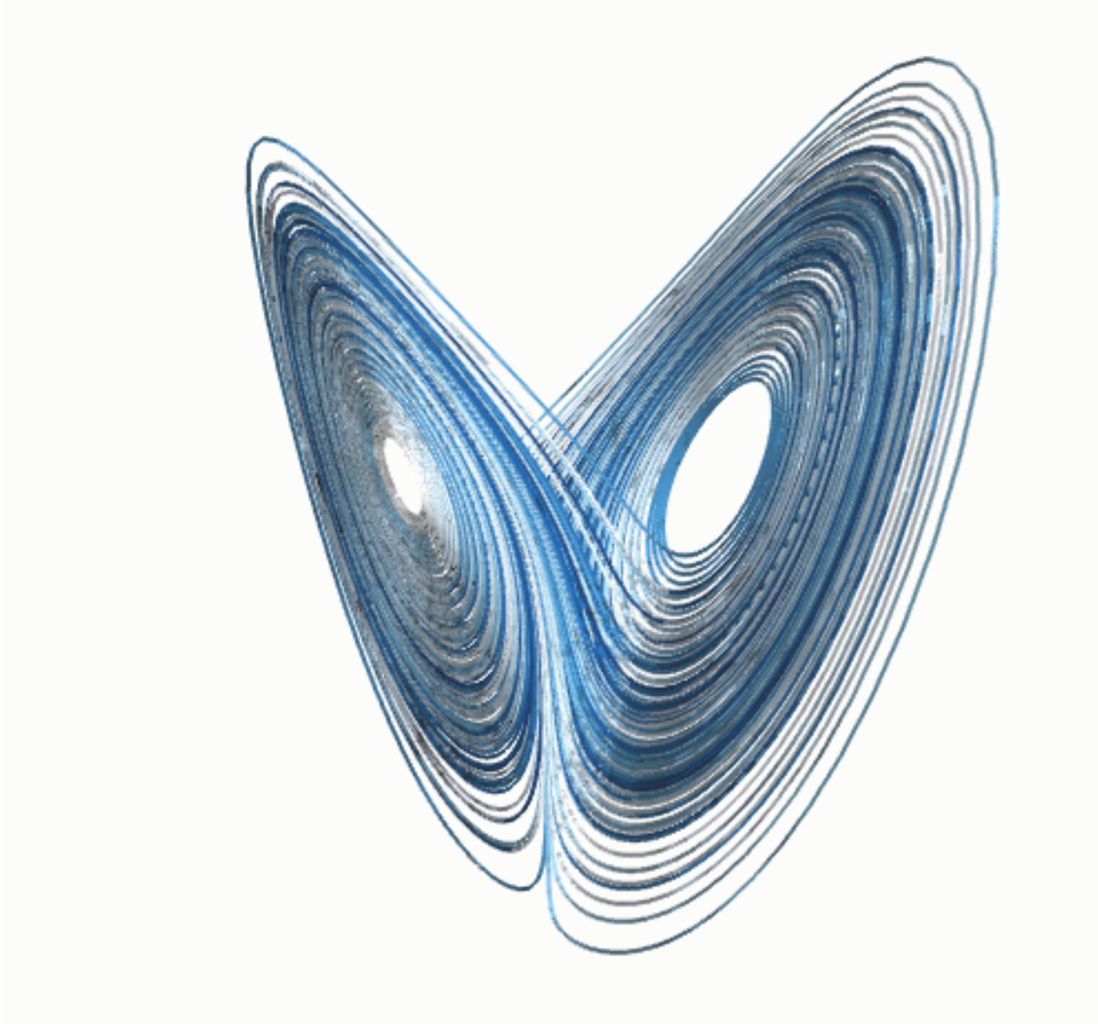
Classical Strange Attractor

Physically closed system, modeled mathematically as autonomous system: neither deterministic (anthropogenic) nor random (natural) forcing.

The attractor is strange, but still fixed in time ~ "irrational" number.

Climate sensitivity ~ change in the average value (first moment) of the coordinates (x, y, z) as a parameter λ changes.



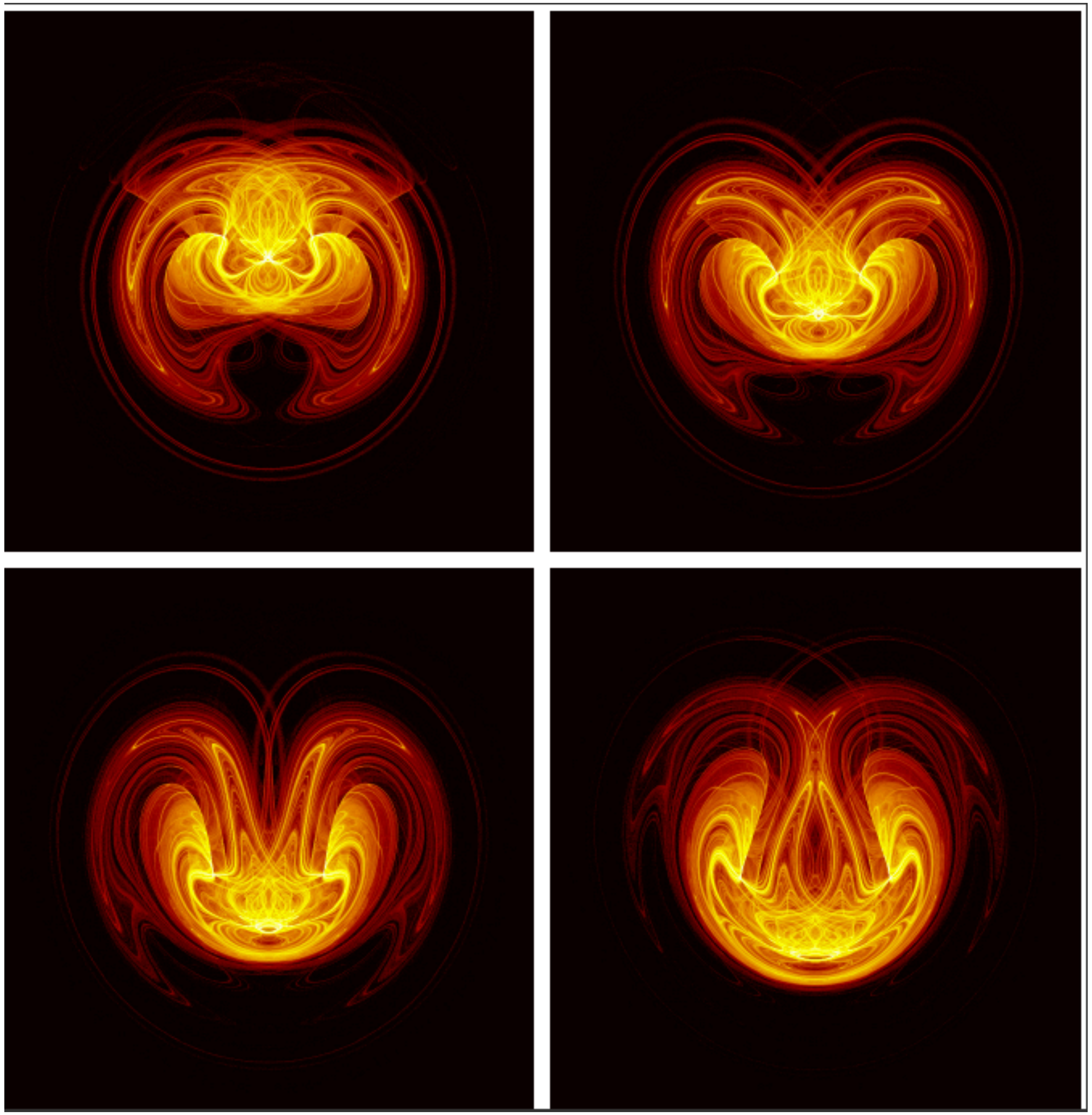
Random Attractor

Physically open system, modeled mathematically as non-autonomous system: allows for deterministic (anthropogenic) as well as random (natural) forcing.

The attractor is "pullback" and evolves in time ~ "imaginary" or "complex" number.

Climate sensitivity ~ change in the statistical properties (first and higher-order moments) of the attractor as one or more parameters (λ, μ, …) change.

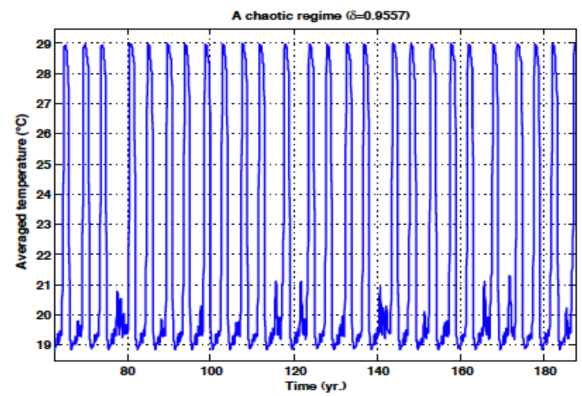
Ghil (*Encyclopedia of Atmospheric Sciences*, 2nd ed., 2012)



Parameter dependence – I

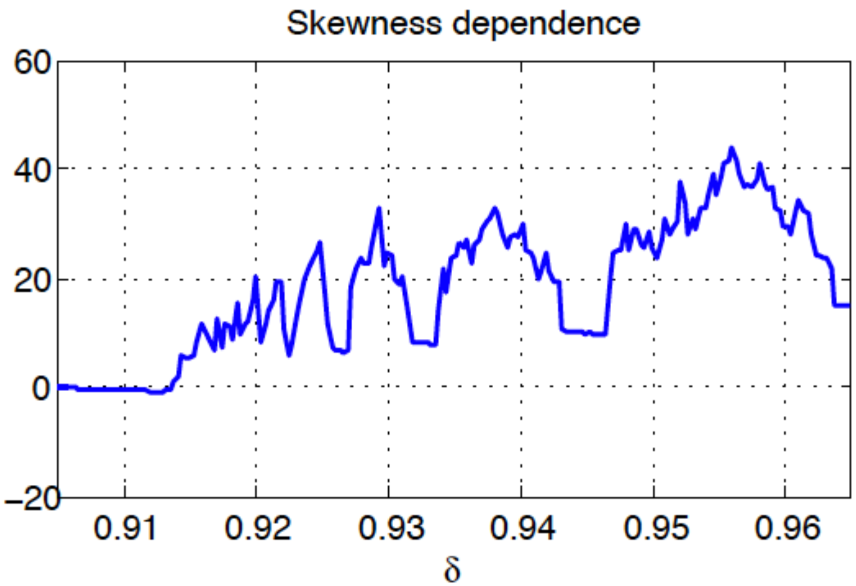
$\delta = 0.9557$

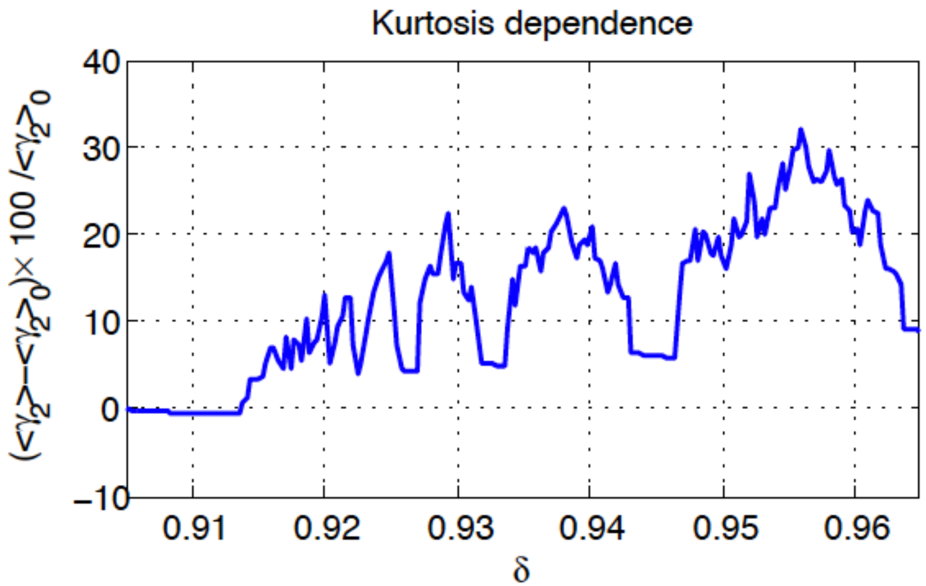
It can be smooth or it can be rough:
Niño-3 SSTs from intermediate coupled model for ENSO (Jin, Neelin & Ghil, 1994, 1996)



Skewness & kurtosis of the SSTs:
time series of 4000 years,

$$\Delta\delta = 3 \cdot 10^{-4}$$





M. Chekroun (work in progress)

Sample measures for an NDDE model of ENSO

The Galanti-Tziperman (GT) model (*JAS*, 1999)

$$\frac{dT}{dt} = -\epsilon_T T(t) - M_0(T(t) - T_{sub}(h(t))),$$

$$h(t) = M_1 e^{-\epsilon_m(\tau_1+\tau_2)} h(t-\tau_1-\tau_2) \\ -M_2\tau_1 e^{-\epsilon_m(\frac{\tau_1}{2}+\tau_2)}\mu(t-\tau_2-\frac{\tau_1}{2})T(t-\tau_2-\frac{\tau_1}{2}) \\ +M_3\tau_2 e^{-\epsilon_m\frac{\tau_2}{2}}\mu(t-\frac{\tau_2}{2})T(t-\frac{\tau_2}{2}).$$

Neutral delay-differential equation (NDDE), derived from Cane-Zebiak and Jin-Neelin models for ENSO: *T* is East-basin SST and *h* is thermocline depth.

Seasonal forcing given by $\mu(t) = 1 + \epsilon\cos(\omega t + \phi)$. The pullback attractor and its invariant measures were computed.

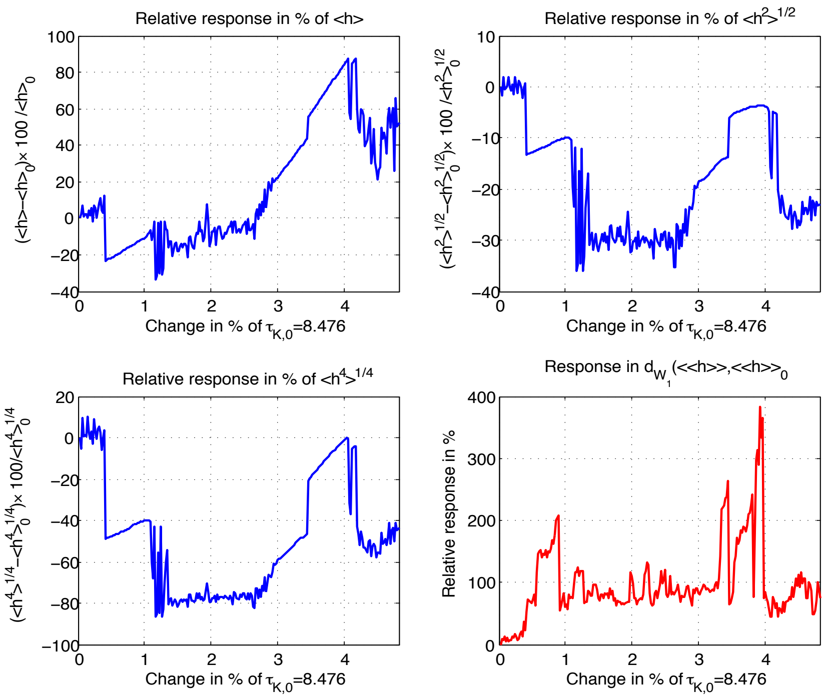


Figure shows the changes in the mean, 2nd & 4th moment of $h(t)$, along with the Wasserstein distance d_W, for changes of 0–5% in the delay parameter $\tau_{\kappa,0}$

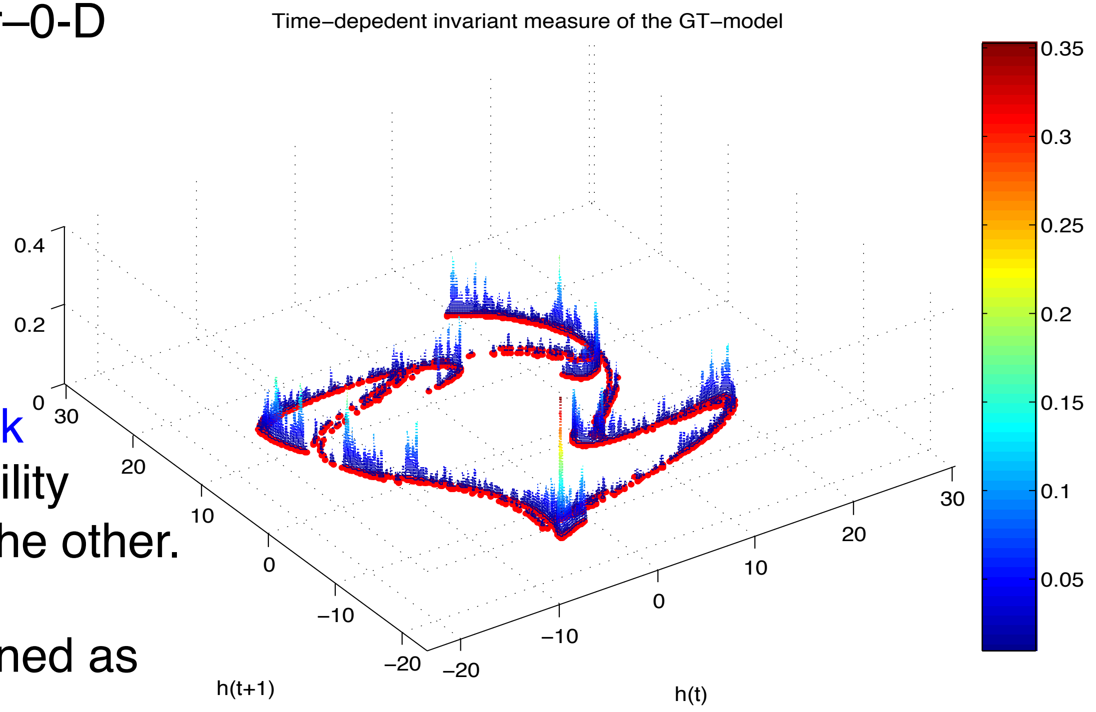
Note intervals of both smooth & rough dependence!

Pullback attractor and invariant measure of the GT model

The time-dependent pullback attractor of the GT model supports an invariant measure $\nu = \nu(t)$, whose density is plotted in 3-D perspective.

The plot is in delay coordinates $h(t+1)$ *vs.* $h(t)$ and the density is highly concentrated along 1-D filaments and, furthermore, exhibits sharp, near–0-D peaks on these filaments.

The Wasserstein distance d_W between one such configuration, at given parameter values, and another one, at a different set of values, is proportional to the work needed to move the total probability mass from one configuration to the other.

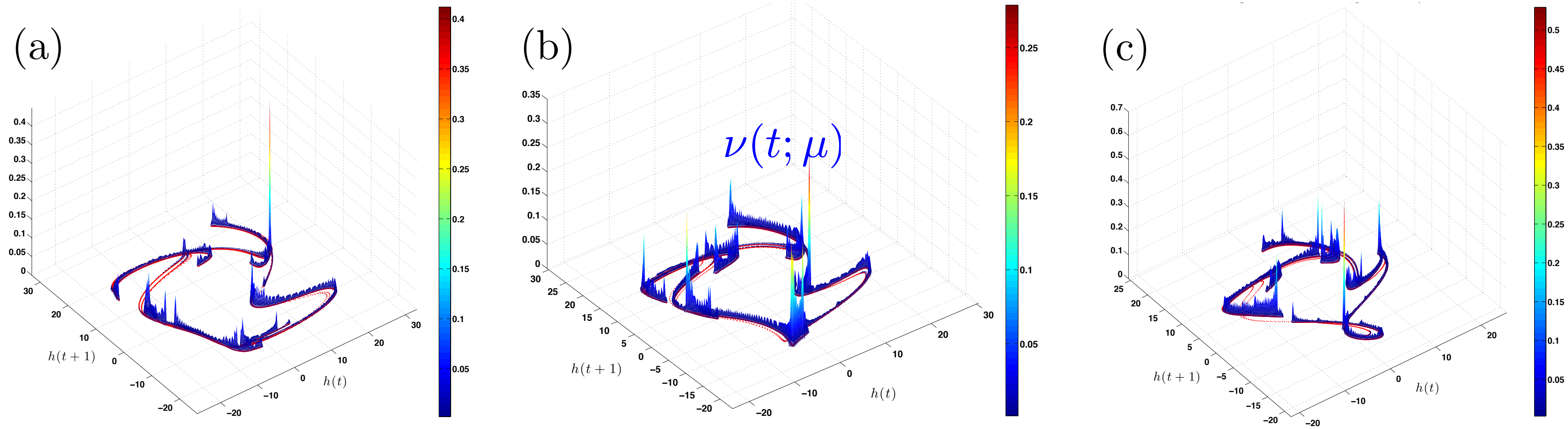


Climate sensitivity γ can be defined as

$$\gamma = \partial d_{\mathrm{W}} / \partial \tau$$

How to define climate sensitivity or, What happens when there's natural variability?

This definition allows us to watch how "the earth moves," as it is affected by both natural and anthropogenic forcing, in the presence of natural variability, which includes both chaotic & random behavior:
chaotic + random behavior:



Clearly the invariant measure $\nu(t;\mu)$ changes in its position (i.e., its support), as well as in its probability density — with time t, as shown here — but also with respect to an arbitrary parameter μ, where $\mu = \tau$ in the present case. Hence, in general,

$$\gamma = \partial d_{\mathrm{W}} / \partial \mu.$$

PBAs of a Low-order Ocean Model Subject to Periodic and Aperiodic Forcing

Stefano Pierini
University of Napoli Parthenope

***Based on joint work with* M.D. Chekroun & M. Ghil**

Please visit these sites for more info.
http://www.atmos.ucla.edu/tcd/
and https://www.researchgate.net/profile/Stefano Pierini

Time-dependent forcing, II

The highly idealized, toy model of the QG, equivalent-barotropic PVE is given by the following system of four quadratically nonlinear ODEs:

$$\dot{\psi}_1 + L_{11}\psi_1 + L_{13}\psi_3 + B_1(\Psi,\Psi) = W_1(t),$$
$$\dot{\psi}_2 + L_{22}\psi_2 + L_{24}\psi_4 + B_2(\Psi,\Psi) = W_2(t),$$
$$\dot{\psi}_3 + L_{33}\psi_3 + L_{31}\psi_1 + B_3(\Psi,\Psi) = W_3(t),$$
$$\dot{\psi}_4 + L_{44}\psi_4 + L_{42}\psi_2 + B_4(\Psi,\Psi) = W_4(t);$$

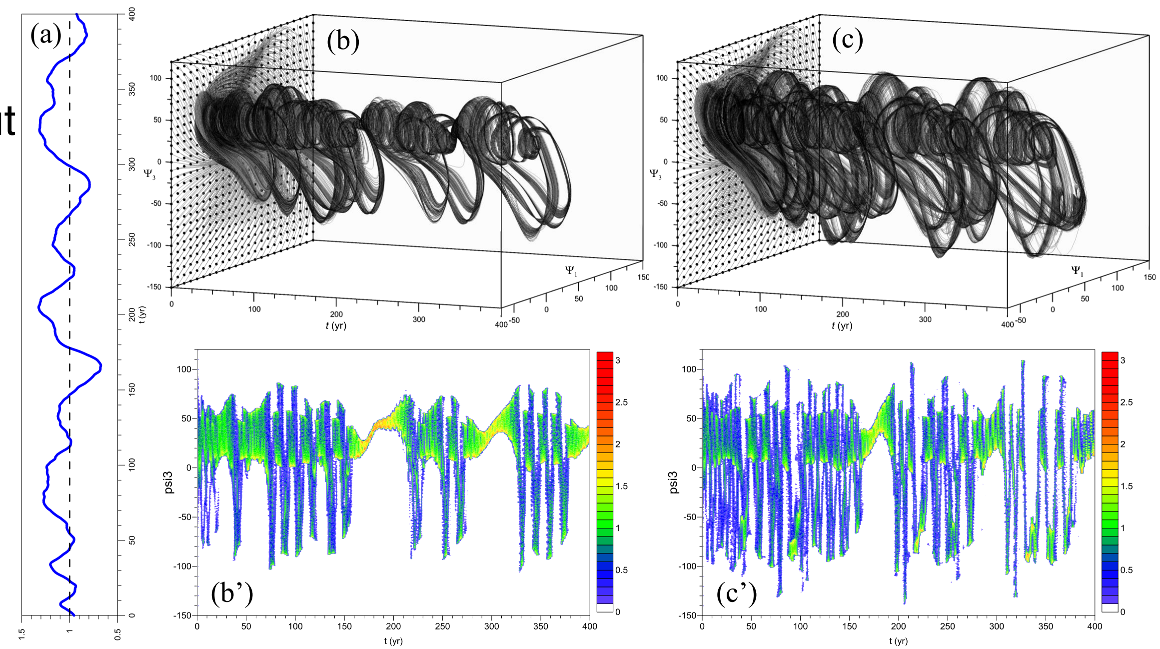
where Ψ denotes the vector $(\psi_1, \psi_2, \psi_3, \psi_4)$ and the bilinear terms B_i are given by

$$\begin{aligned}
B_1(\Psi,\Psi) &= 2J_{112}\psi_1\psi_2 + 2J_{114}\psi_1\psi_4 + 2J_{134}\psi_3\psi_4,\\
B_2(\Psi,\Psi) &= J_{211}\psi_1^2 + J_{222}\psi_2^2 + J_{233}\psi_3^2\\
&\quad + 2J_{213}\psi_1\psi_3 + 2J_{224}\psi_2\psi_4,\\
B_3(\Psi,\Psi) &= 2J_{314}\psi_1\psi_4 + 2J_{323}\psi_2\psi_3 + 2J_{334}\psi_3\psi_4,\\
B_4(\Psi,\Psi) &= J_{411}\psi_1^2 + J_{422}\psi_2^2 + J_{433}\psi_3^2 + J_{444}\psi_4^2\\
&\quad + 2J_{413}\psi_1\psi_3 + 2J_{424}\psi_2\psi_4.
\end{aligned}$$

Time-dependent forcing, III

- The quadratic terms are conservative and the linear terms are weakly dissipative, while the system is unstable for reasonable parameter values.
- For autonomous systems, we know that these properties can lead to chaotic solutions that live on a strange attractor.
- Here they lead to the existence of a pullback attractor (PBA).

Time-dependent forcing at far left + ensemble of 644 orbits starting from the same subset *Γ* but with different parameter values: left panels, close to periodic; right panels, fully chaotic; lower panels, time series of ψ_3. Time interval is 400 years.

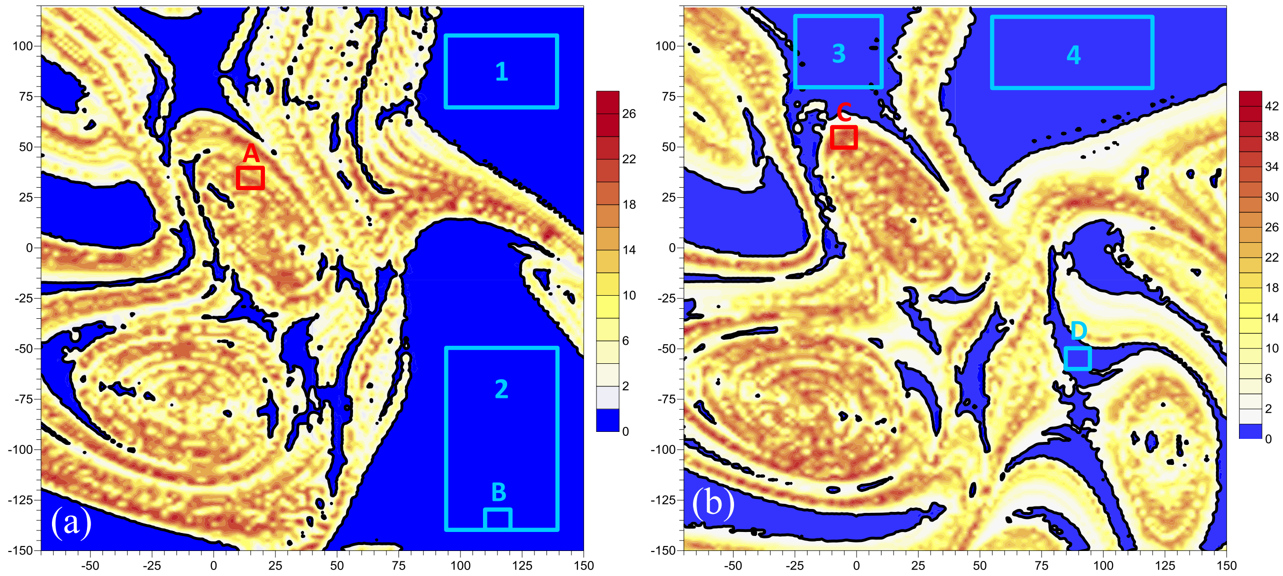


Pierini, Ghil & Chekroun (*J. Clim.*, 2016)

Time-dependent forcing, IV

- There are strong numerical indications, along with theoretical justifications, that multiple PBAs are present within a global attractor.
- Moreover, preliminary numerical results suggest that the basin boundaries between two attractors are fractal.

Measure of divergence of trajectories for each initial point in the (ψ_1, ψ_3)-plane in the remote past: blue indicates stability; parameter values (left) and (right) are the same as in the previous figure.

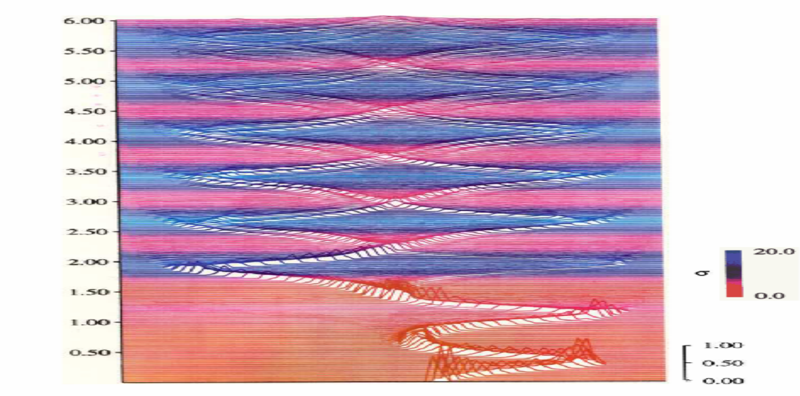


Pierini, Ghil & Chekroun
(*J. Clim.*, 2016)

Outline

- The IPCC process: results and uncertainties
- Natural climate variability as a source of uncertainties
  - sensitivity to initial data ➔ error growth
  - sensitivity to model formulation ➔ see below!
- Uncertainties and how to fix them
  - structural stability and other kinds of robustness
  - non-autonomous and random dynamical systems (NDDS & RDS)
- Two illustrative examples
  - the Lorenz convection model
  - an El Niño–Southern Oscillation (ENSO) model
- Linear response theory and climate sensitivity
- Conclusions and references
  - natural variability and anthropogenic forcing: the "grand unification"
  - selected bibliography

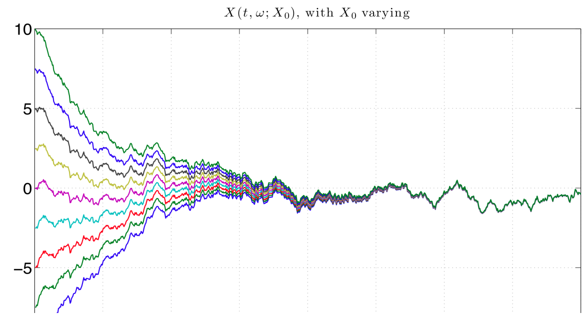
Yet another (grand?) unification



Lorenz (*JAS*, 1963)

Climate is deterministic and autonomous,
but highly nonlinear.

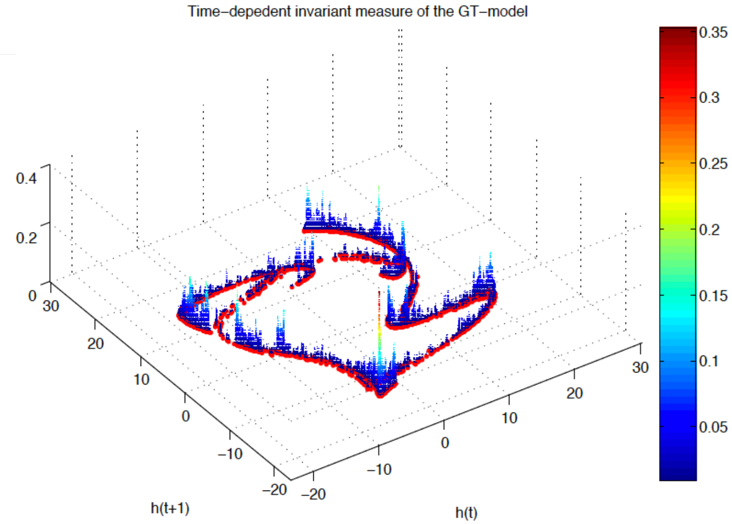
Trajectories diverge exponentially,
forward asymptotic PDF is multimodal.



Hasselmann (*Tellus*, 1976)

Climate is stochastic and noise-driven,
but quite linear.

Trajectories decay back to the mean,
forward asymptotic PDF is unimodal.



Grand unification (?)

**Climate is deterministic + stochastic,
as well as highly nonlinear.**

**Internal variability and forcing interact
strongly, change and sensitivity
refer to both mean and higher moments.**

Concluding remarks – NDS and RDS

Summary

- A change of paradigm from **closed, autonomous systems** to **open, non-autonomous ones.**
- Pullback and random attractors are
  (i) spectacular, (ii) useful, and
  (iii) just starting to be explored for climate applications.

Work in progress

- Applications to intermediate models and GCMs.
- Implications for climate sensitivity — higher moments, ExEv's.
- Implications for predictability?

Some general references

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Reserve slides

Crisis of strange PBAs in a delay differential model of the El Niño–Southern Oscillation

Mickael Chekroun
University of California, Los Angeles

***Based on joint work with* M. Ghil and J.D. Neelin**

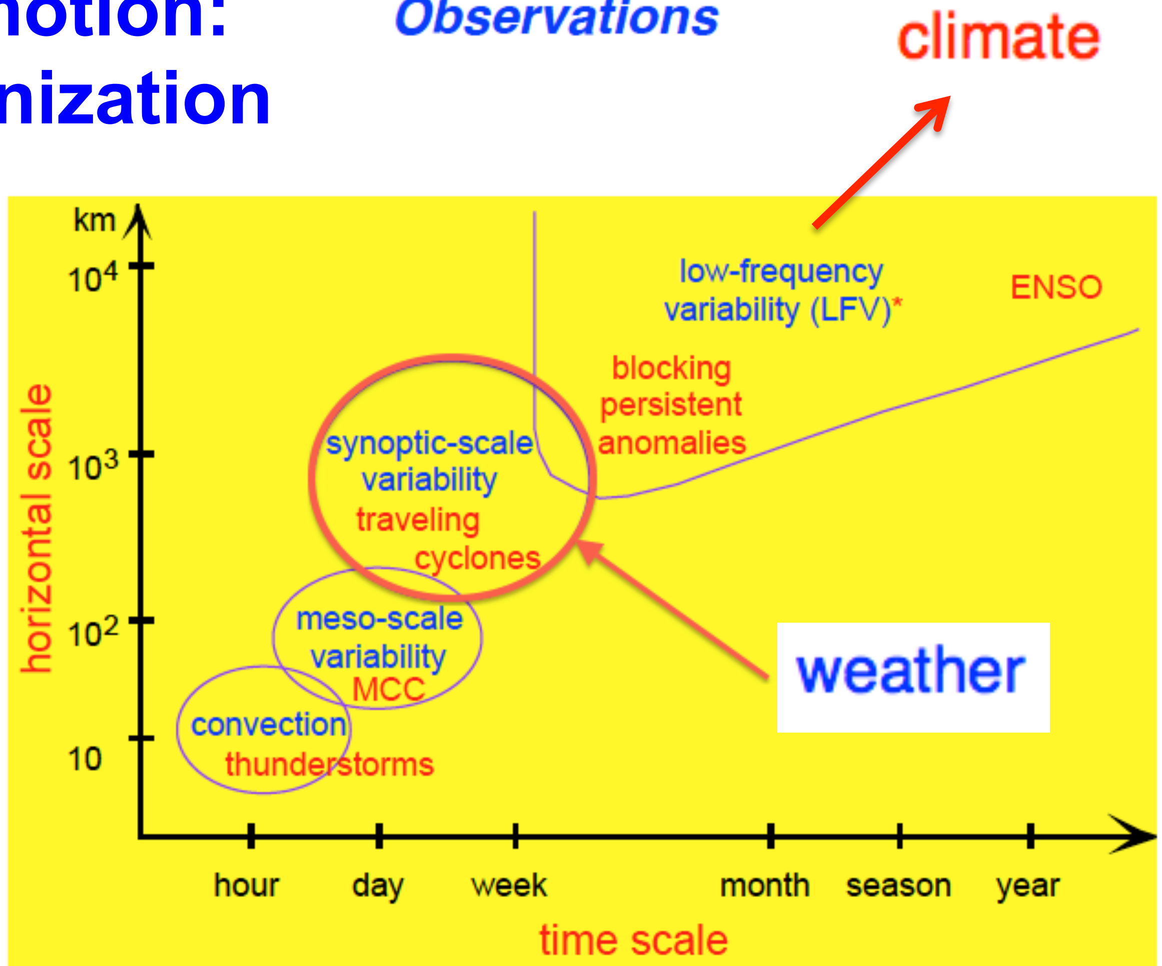
ENS

Please visit these sites for more info.
http://www.atmos.ucla.edu/tcd/
and https://www.researchgate.net/profile/Mickael_Chekroun

Multiple scales of motion: Space-time organization

Observations

- The most active scales lie along a diagonal in this space *vs.* time plot.
- Why this is so is far from clear as of now.
- We'll deal with weather first, then climate.



N.B. A **high-variability ridge** lies close to the diagonal of the plot (*cf.* also Fraedrich & Böttger, 1978, *JAS*)

* LFV ≅ 10–100 days (intraseasonal)

nature 28 March 1991

letters to nature

Nature **350**, 324 - 327 (1991); doi:10.1038/350324a0

Interdecadal oscillations and the warming trend in global temperature time series

M. Ghil & R. Vautard

THE ability to distinguish a warming trend from natural variability is critical for an understanding of the climatic response to increasing greenhouse-gas concentrations. Here we use singular spectrum analysis[1] to analyse the time series of global surface air tem-peratures for the past 135 years[2], allowing a secular warming trend and a small number of oscillatory modes to be separated from the noise. The trend is flat until 1910, with an increase of 0.4 °C since then. The oscillations exhibit interdecadal periods of 21 and 16 years, and interannual periods of 6 and 5 years. The interannual oscillations are probably related to global aspects of the El Niño-Southern Oscillation (ENSO) phenomenon[3]. The interdecadal oscillations could be associated with changes in the extratropical ocean circulation[4]. The oscillatory components have combined (peak-to-peak) amplitudes of 0.2 °C, and therefore limit our ability to predict whether the inferred secular warming trend of 0.005 $°Cyr^{-1}$ will continue. This could postpone incontrovertible detection of the greenhouse warming signal for one or two decades.

Can we, nonlinear dynamicists, help?

The uncertainties
might be ***intrinsic***,
rather than mere
"tuning problems"

If so, maybe
stochastic structural stability could help!

Might fit in nicely with
recent taste for
"stochastic
parameterizations"

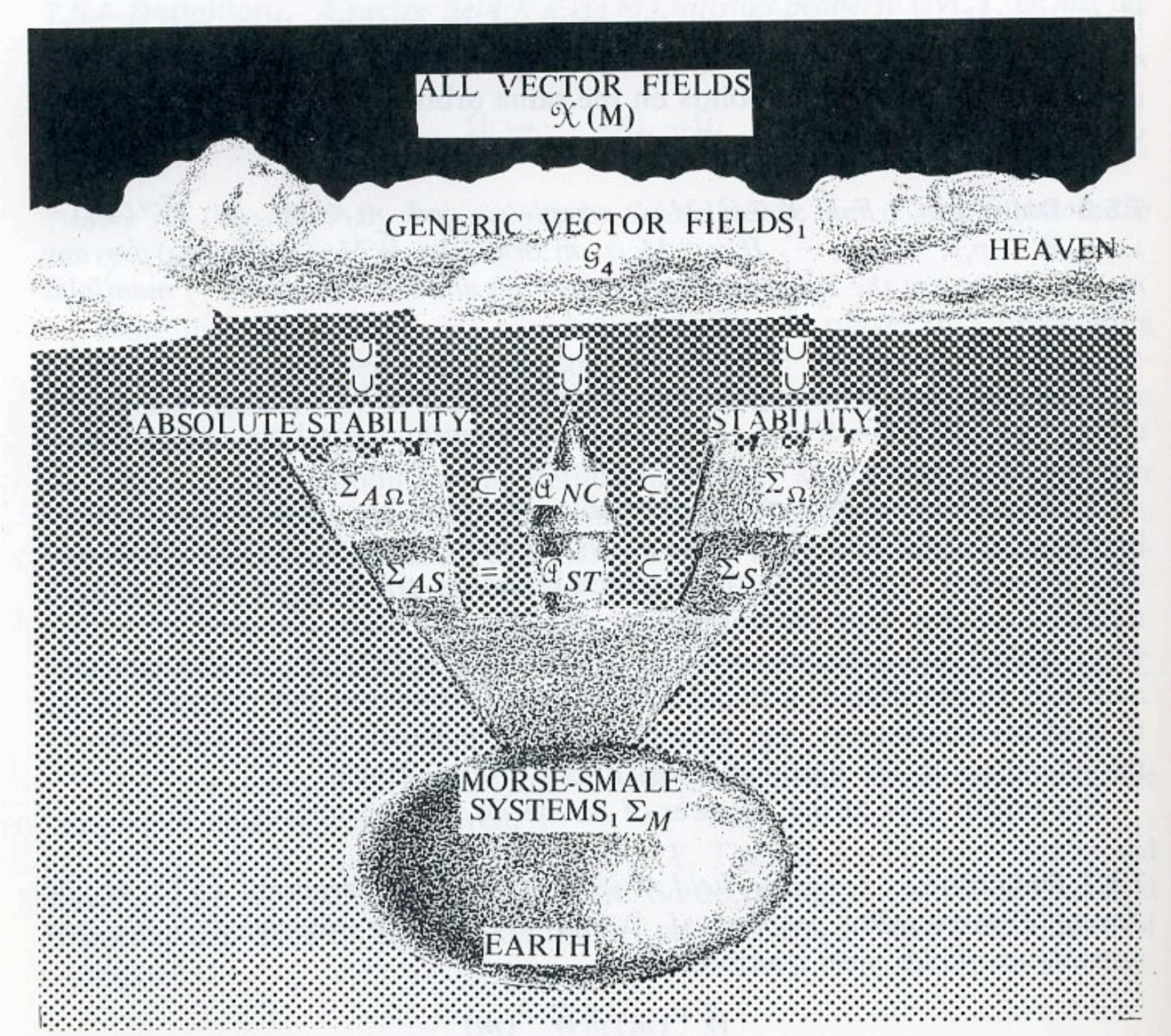
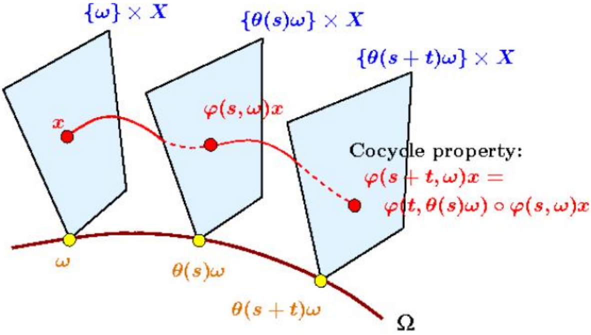


Figure 7.5-1. The three towers of differentiable dynamics.

The DDS dream of structural stability (from *Abraham & Marsden*, 1978)



- φ **is a random dynamical system (RDS)**
- $\Theta(t)(x,\omega) = (\theta(t)\omega, \varphi(t,\omega)x)$ **is a flow on the bundle**

Global warming and "global weirding"

"CLIMATE STRANGE
FORGET GLOBAL WARMING—AND
GET READY for GLOBAL WEIRDING
BY BRYAN WALSH"

TIME MAGAZINE, Dec. 29, 2014 – Jan. 5, 2015

"***The New Rule***: For the next few (?) years, global warming will lead to colder, more brutal winters."

- Oh, thank you for the latest prediction from a science journalist — based on interesting but still rather tentative, & hotly debated, suggestions from a few media-loving (& vice-versa) researchers.

- And if this is so certain, why wasn't it predicted by IPCC(*) and other models BEFORE it happened?

(*) Intergovernmental Panel on Climate Change

Transitions Between Blocked and Zonal Flows
in a Barotropic Rotating Annulus with Topography

Zonal Flow
13–22 Dec. 1978

Blocked Flow
10–19 Jan. 1963

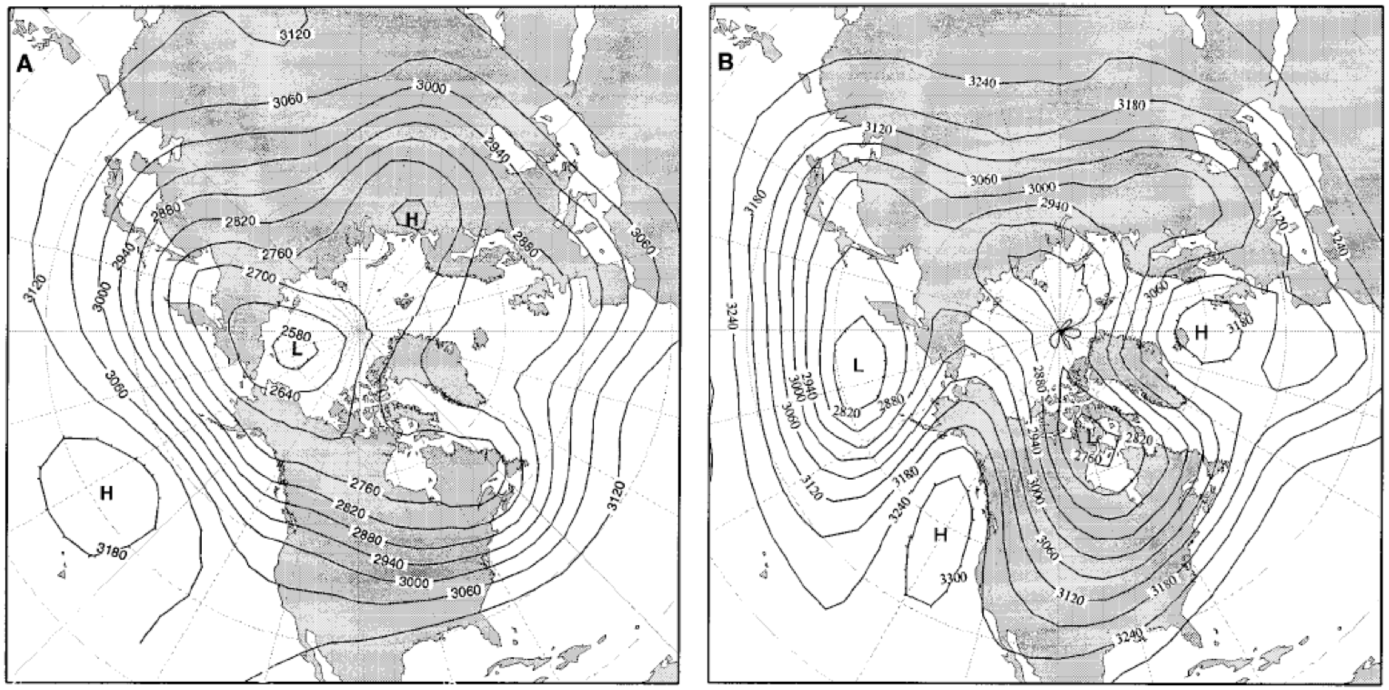
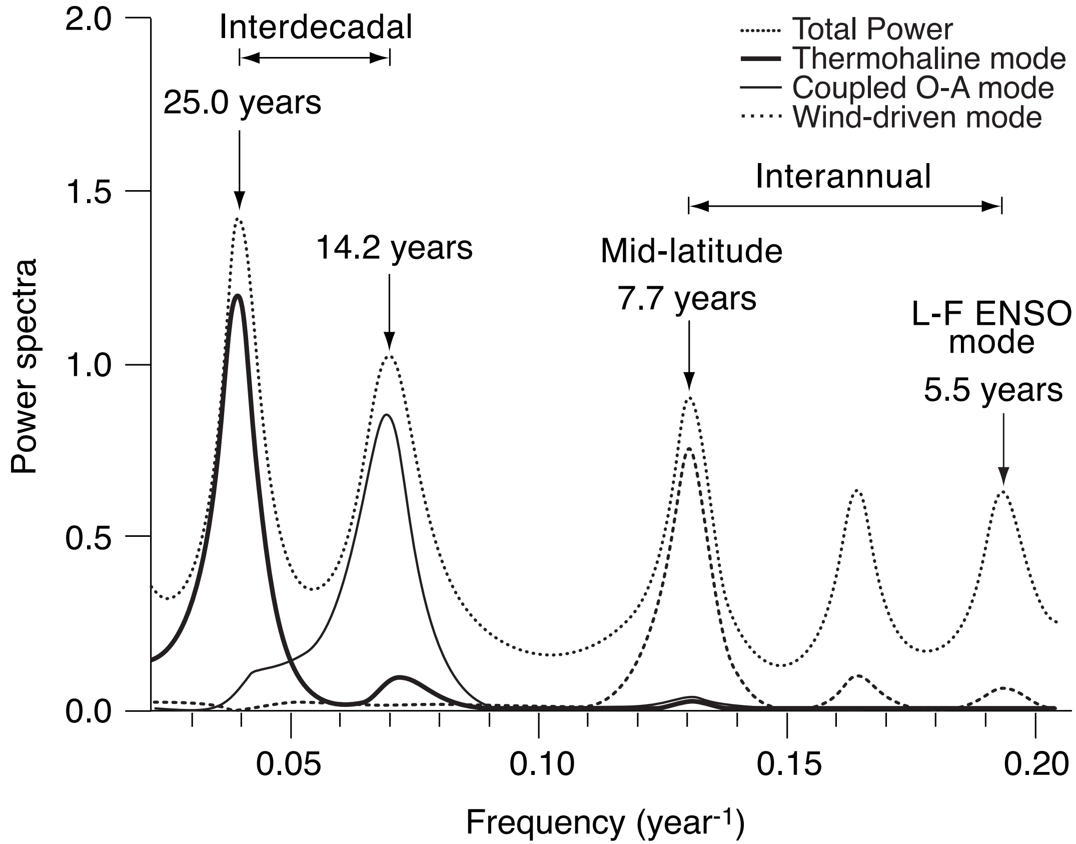


Fig. 1. Atmospheric pictures of (**A**) zonal and (**B**) blocked flow, showing contour plots of the height (m) of the 700-hPa (700 mbar) surface, with a contour interval of 60 m for both panels. The plots were obtained by averaging 10 days of twice-daily data for (A) 13 to 22 December 1978 and (B) 10 to 19 January 1963; the data are from the National Oceanic and Atmospheric Administration's Climate Analysis Center. The nearly zonal flow of (A) includes quasi-stationary, small-amplitude waves (32). Blocked flow advects cold Arctic air southward over eastern North America or Europe, while decreasing precipitation in the continent's western part (26).

Weeks, Tian, Urbach, Ide, Swinney, & Ghil (*Science*, 1997)

SSA (prefilter) + (low-order) MEM

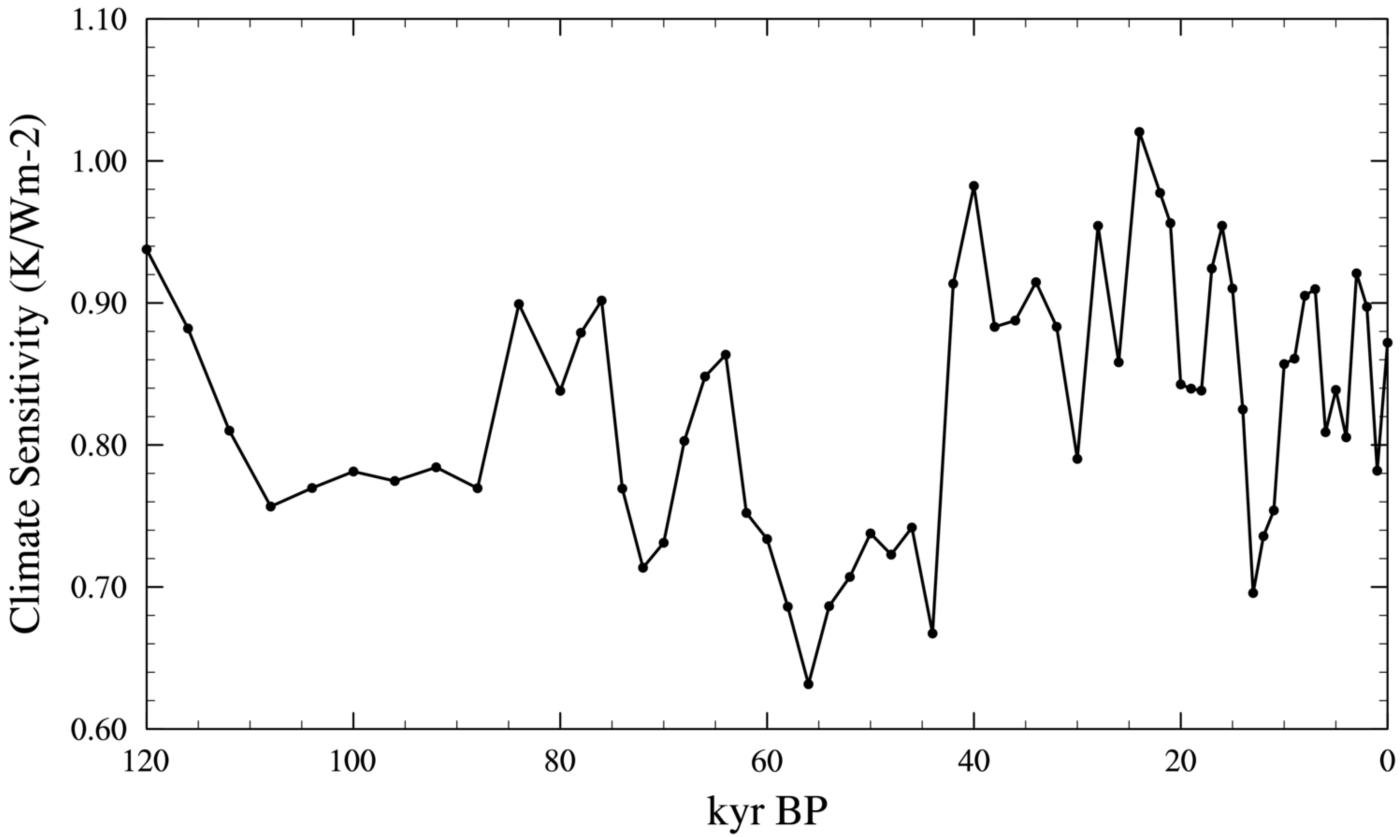
- "Stack" spectrum



In good agreement with MTM peaks of **Ghil & Vautard (1991, *Nature*)** for the Jones *et al.* (1986) temperatures & stack spectra of Vautard *et al.* (1992, *Physica D*) for the IPCC "consensus" record (both global), to wit 26.3, 14.5, 9.6, 7.5 and 5.2 years.

Peaks at 27 & 14 years also in Koch sea-ice index off Iceland (Stocker & Mysak, 1992), etc.
Plaut, Ghil & Vautard (1995, *Science*)

Modeled Climate Sensitivity

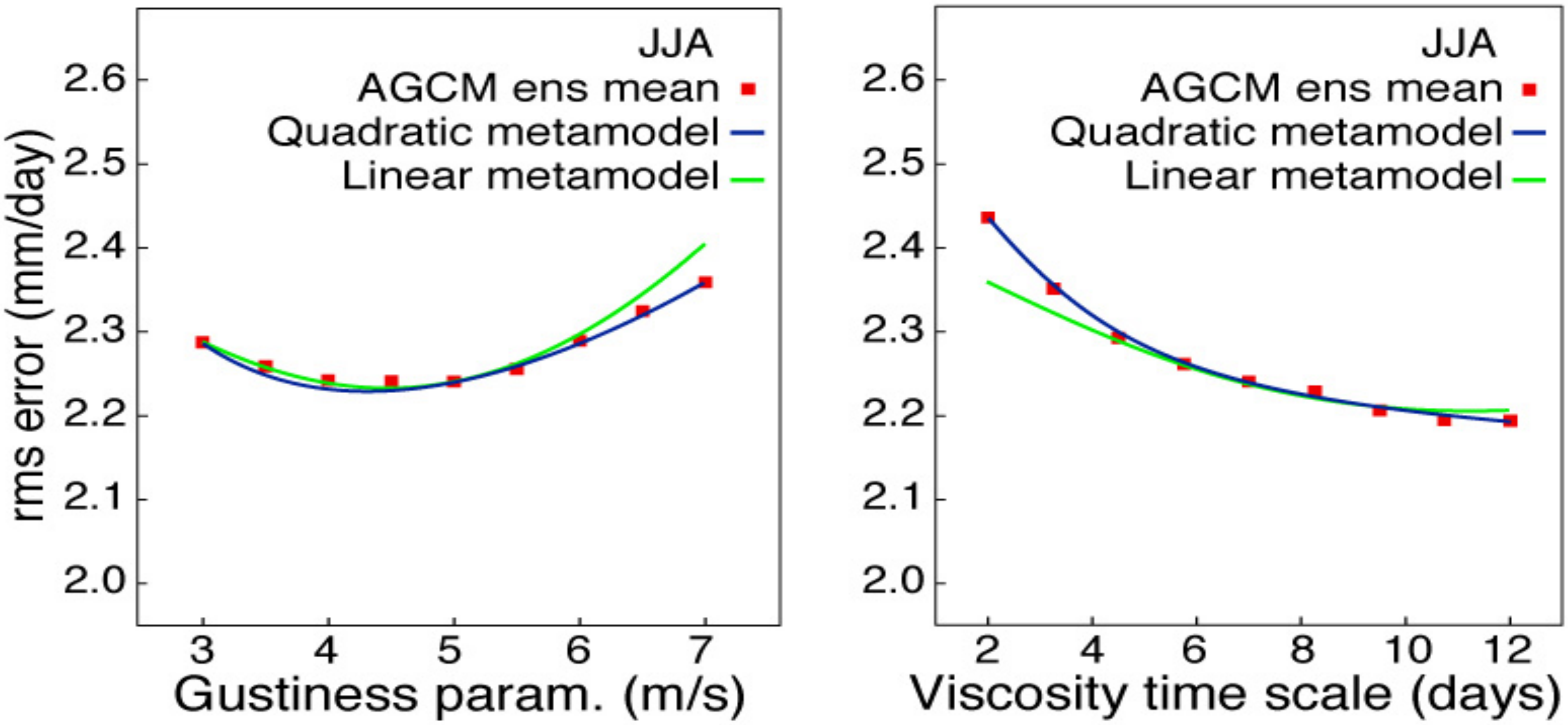


Climate sensitivity as estimated from a series of "snapshot" simulations of paleoclimate using HadCM3.

Courtesy of Paul J. Valdes

Parameter dependence – II

When it is smooth, one can optimize a GCM's single-parameter dependence



ICTP AGCM (Neelin, Bracco, Luo, McWilliams & Meyerson, *PNAS*, 2010)

Concluding remarks, II – Climate change & climate sensitivity

What do we know?

- It's getting warmer.
- We do contribute to it.
- So we should act as best we know and can!

What do we know less well?

- By how much?
  – Is it getting warmer …
  – Do we contribute to it …
- How does the climate system (atmosphere, ocean, ice, etc.) really work?
- How does natural variability interact with anthropogenic forcing?

What to do?

- Better understand the system and its forcings.
- Explore the models', and the system's, robustness and sensitivity
  – stochastic structural and statistical stability!
  – linear response = response function + susceptibility function!!

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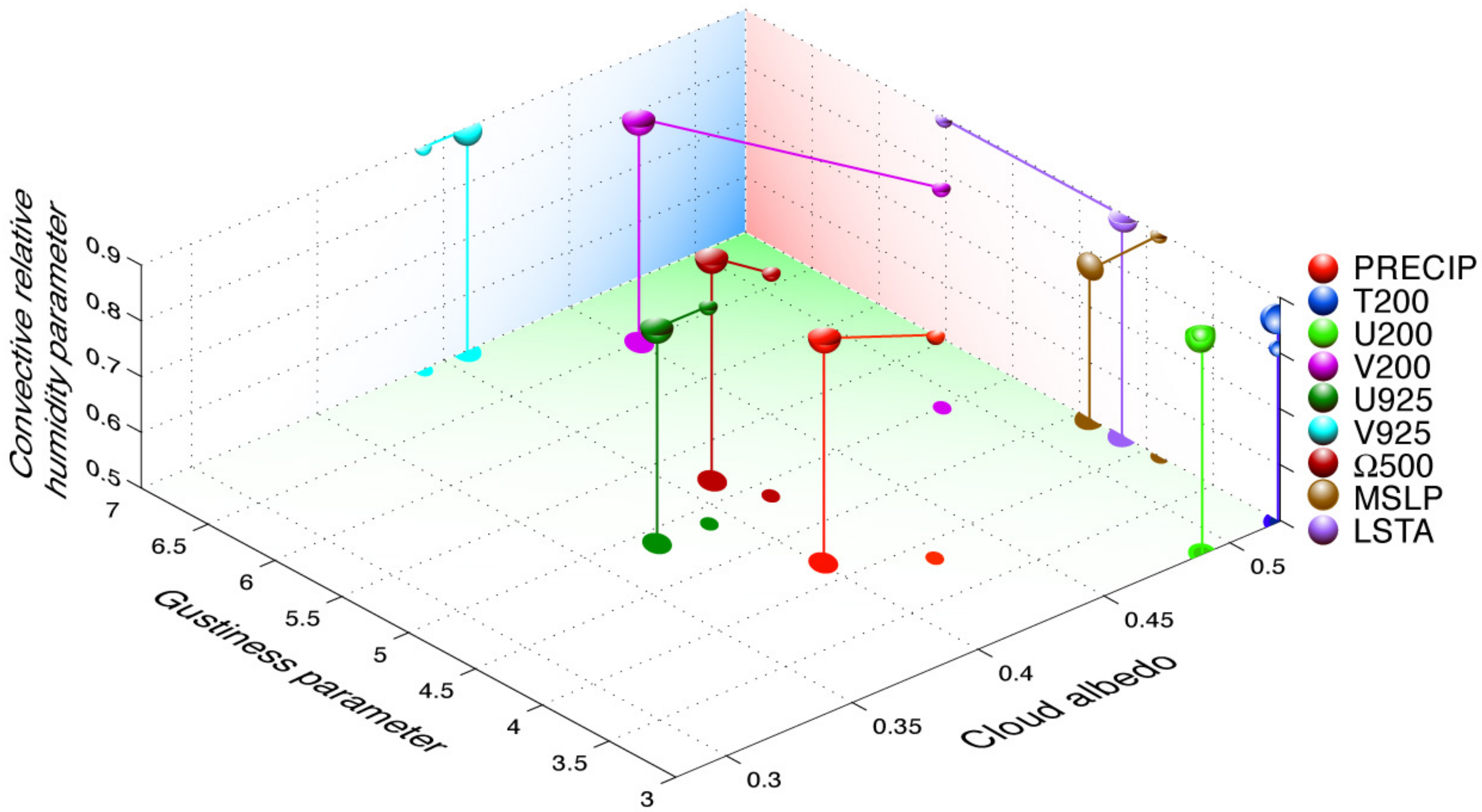
Parameter dependence – III

Multi-objective algorithms avoid arbitrary weighting of criteria
in a unique cost function:



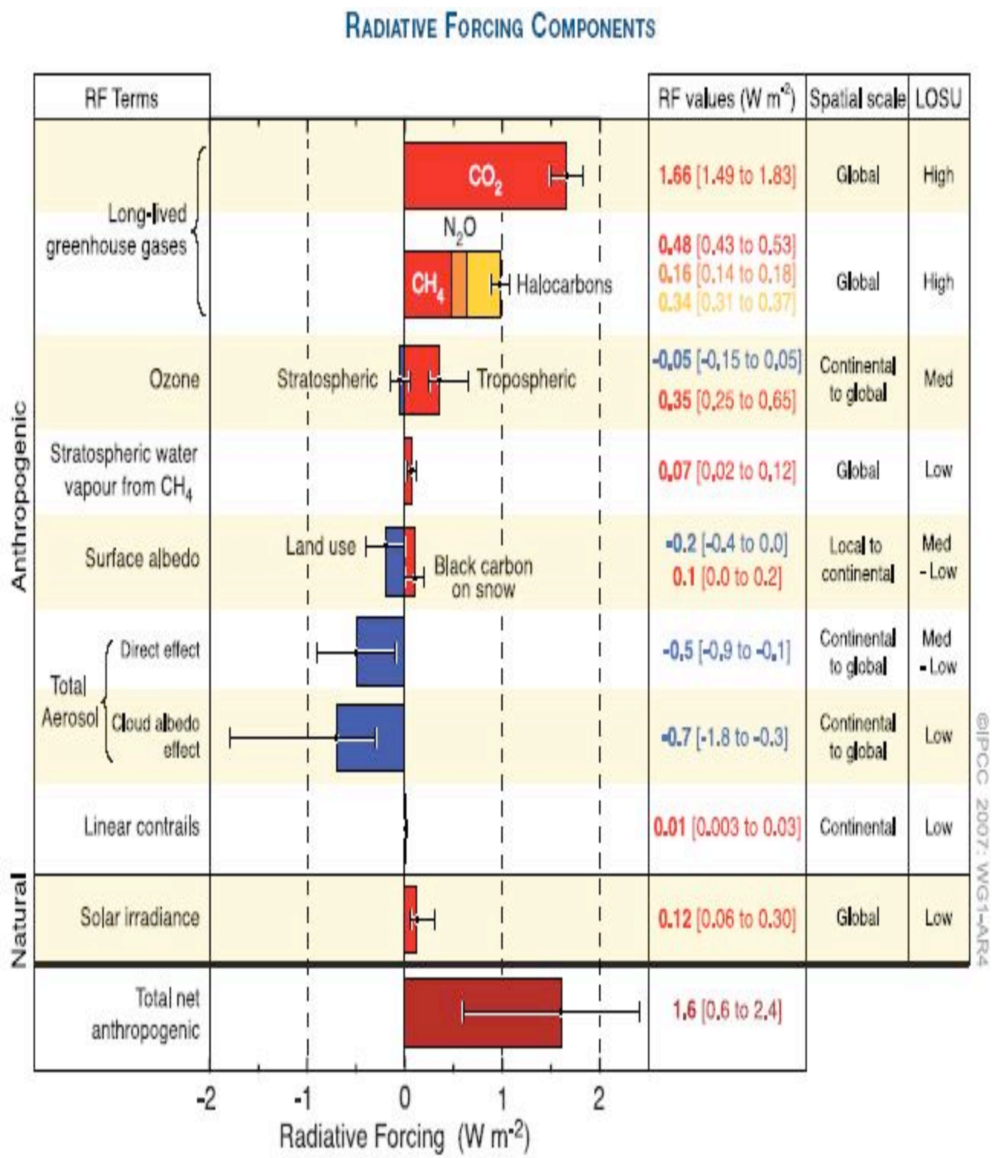
Optimization algorithms that are $\mathcal{O}(N)$ and $\mathcal{O}(N^2)$, rather than $\mathcal{O}(S^N)$, where N is the number of parameters and S is the sampling density.

ICTP AGCM (Neelin, Bracco, Luo, McWilliams & Meyerson, *PNAS*, 2010)

GHGs rise!

It's gotta do with us, at least a bit, ain't it?

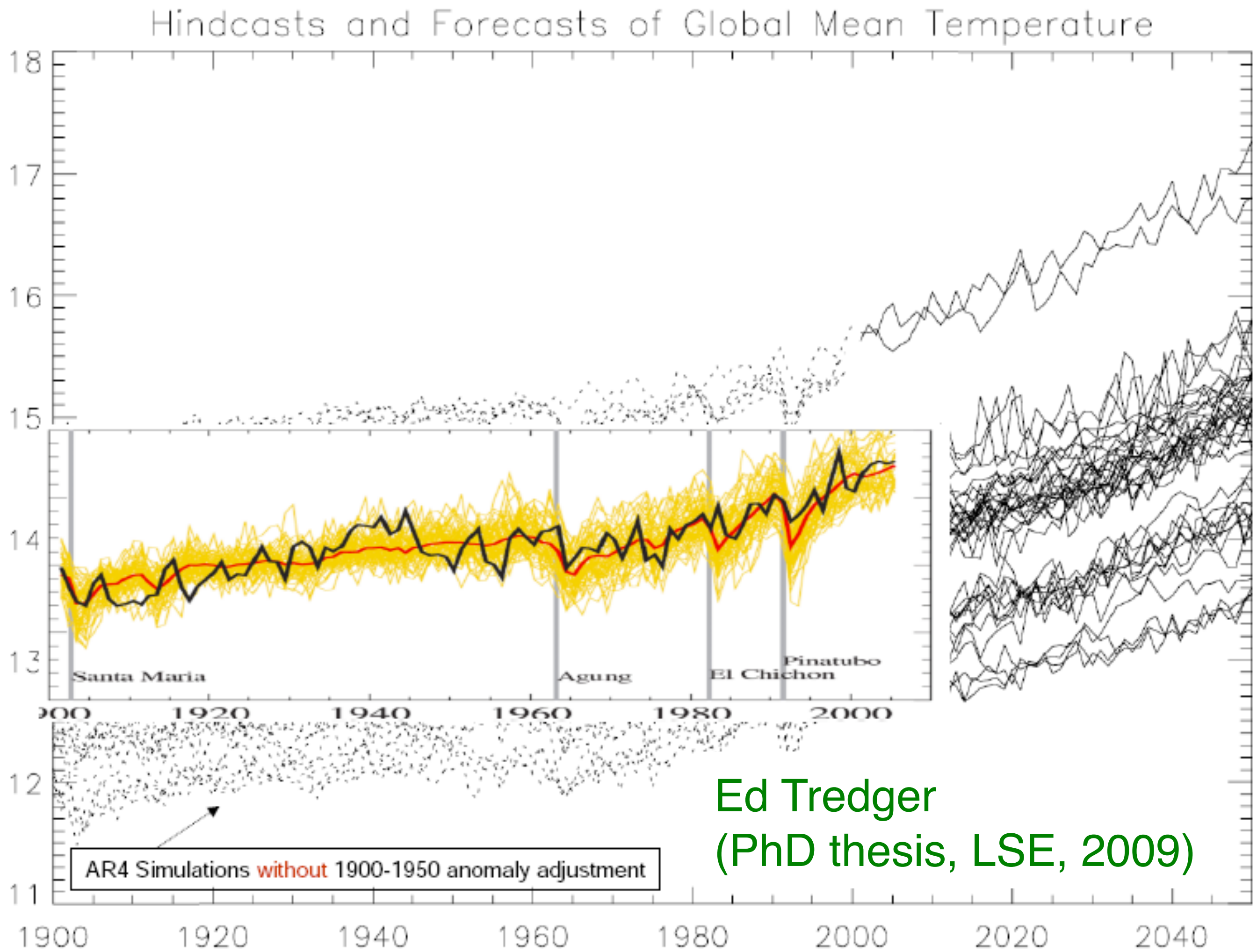
But just how much?



IPCC (2007)

www.lseca

AR4 adjustment of 20th century simulation



Ed Tredger
(PhD thesis, LSE, 2009)

Grantham Research Institute on Climate Change and the Environment

L.A. ("Lenny") Smith (2009) personal communication

Remarks

- We've just shown that:

$$|x(t,s;x_0) - a(t)| \underset{s\to-\infty}{\longrightarrow} 0 \text{ ; for every } t \text{ fixed,}$$

  and for all initial data x_0, with $a(t) = \frac{\sigma}{\alpha}(t - 1/\alpha)$.
- We've just encountered the concept of pullback attraction; here $\{a(t)\}$ is the pullback attractor of the system (1).
- What does it mean physically?
  The pullback attractor provides a way to assess an asymptotic regime at time t — the time at which we observe the system — for a system starting to evolve from the remote past s, $s << t$.
- This asymptotic regime evolves with time: it is a dynamical object.
- Dissipation now leads to a dynamic object rather than to a static one, like the strange attractor of an autonomous system.

Remarks

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This theory is the counterpart for randomly forced dynamical systems (RDS) of the *geometric theory* of ordinary differential equations (ODEs). It allows one to treat stochastic differential equations (SDEs) — and more general systems driven by noise — as flows in (phase space)$\times$(probability space).

SDE$\sim$ODE, RDS$\sim$DDS, L. Arnold (1998)$\sim$V.I. Arnol'd (1983).

Setting:

(i) A phase space X. **Example**: $\mathbb{R}^n$.

(ii) A probability space $(\Omega, \mathcal{F}, \mathbb{P})$. **Example**: The Wiener space $\Omega = C_0(\mathbb{R}; \mathbb{R}^n)$ with Wiener measure $\mathbb{P}$.

(iii) A model of the noise $\theta(t) : \Omega \to \Omega$ that preserves the measure $\mathbb{P}$, i.e. $\theta(t)\mathbb{P} = \mathbb{P}$; θ is called the driving system.
Example: $W(t, \theta(s)\omega) = W(t + s, \omega) - W(s, \omega)$;
it starts the noise at s instead of $t = 0$.

(iv) A mapping $\varphi : \mathbb{R} \times \Omega \times X \to X$ with the cocycle property.
Example: The solution operator of an SDE.

Chekroun, Simonnet and Ghil, 2011

Timmerman & Jin (*Geophys. Res. Lett.*, 2002) have derived the following low-order, tropical-atmosphere–ocean model. The model has three variables: thermocline depth anomaly h, and SSTs T_1 and T_2 in the western and eastern basin.

$$\begin{aligned} \dot{T}_1 &= -\alpha(T_1 - T_r) - \frac{2\varepsilon u}{L}(T_2 - T_1), \\ \dot{T}_2 &= -\alpha(T_2 - T_r) - \frac{w}{H_m}(T_2 - T_{sub}), \\ \dot{h} &= r(-h - bL\tau/2). \end{aligned}$$

The related diagnostic equations are:

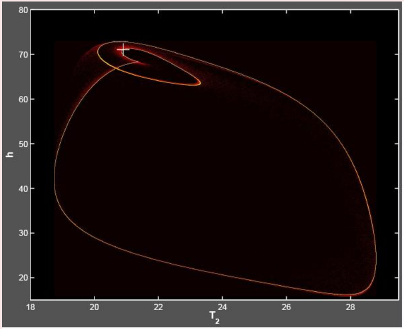
$$\begin{aligned} T_{sub} &= T_r - \frac{Tr - T_{r0}}{2}[1 - \tanh(H + h_2 - z_0)/h^*] \\ \tau &= \frac{a}{\beta}(T_1 - T_2)[\xi_t - 1]. \end{aligned}$$

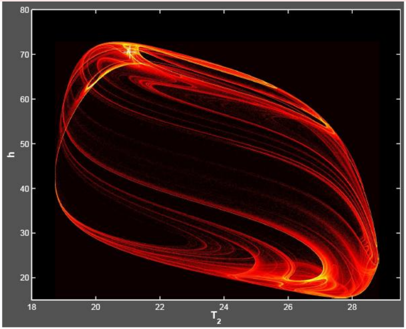
- τ: the wind stress anomalies, $w = -\beta\tau/H_m$: the equatorial upwelling.
- $u = \beta L\tau/2$: the zonal advection, T_{sub}: the subsurface temperature.

Wind stress bursts are modeled as white noise ξ_t of variance σ, and ε measures the strength of the zonal advection.

Random attractors: the frozen statistics

Random Shil'nikov horseshoes

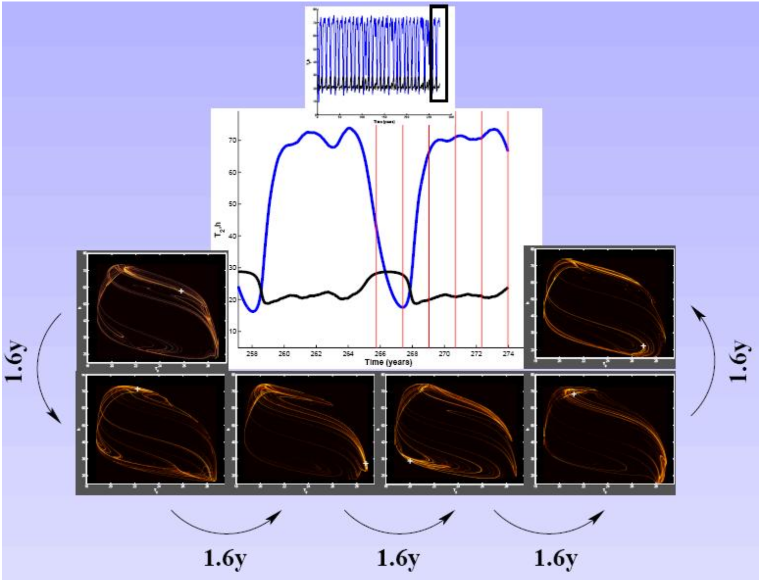




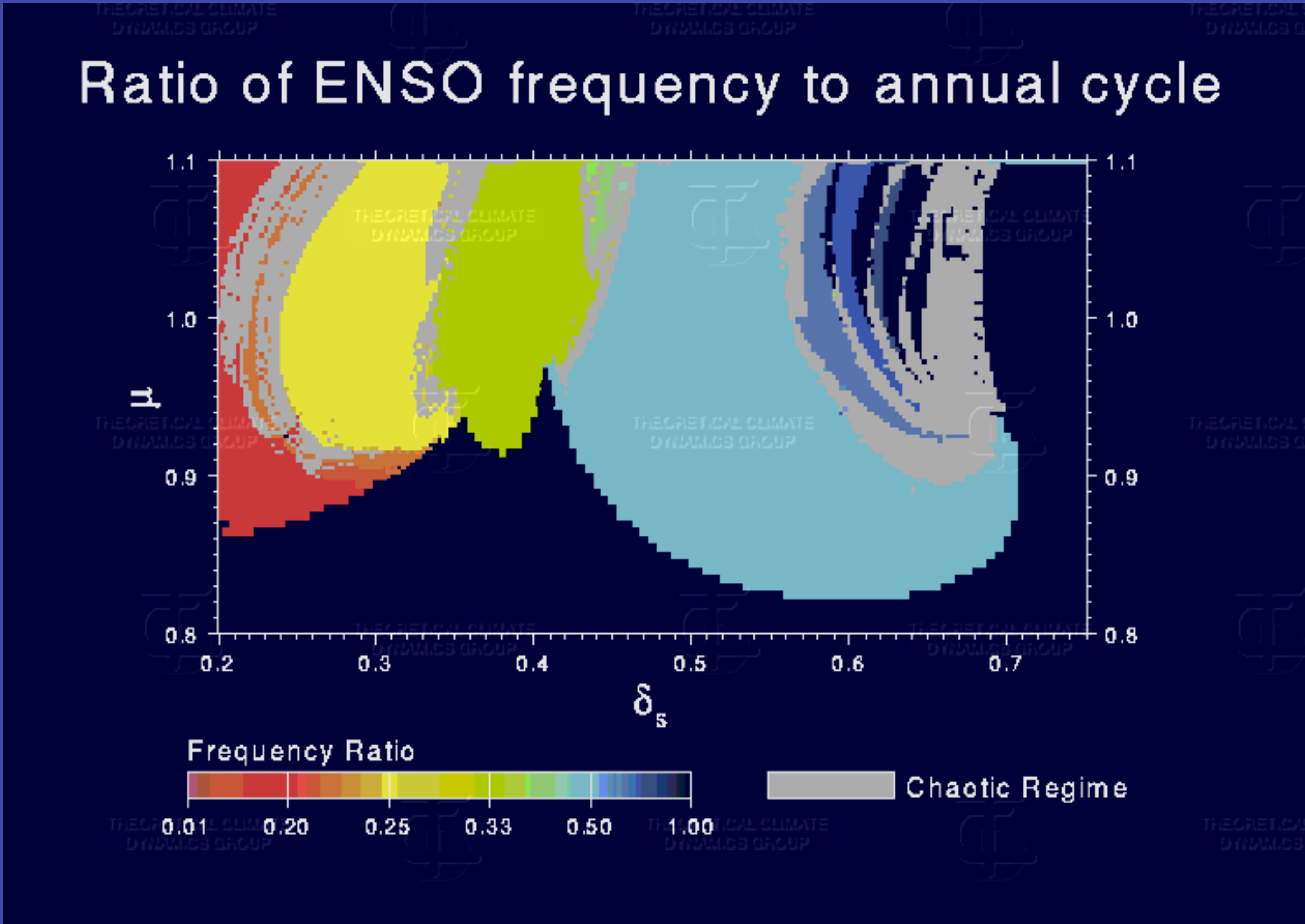
σ=0.005 σ=0.05

- Horseshoes can be noise-excited,
  left: a weakly-perturbed limit cycle, right: the same with larger noise.
- Golden: most frequently-visited areas; white 'plus' sign: most visited.

An episode in the random's attractor life



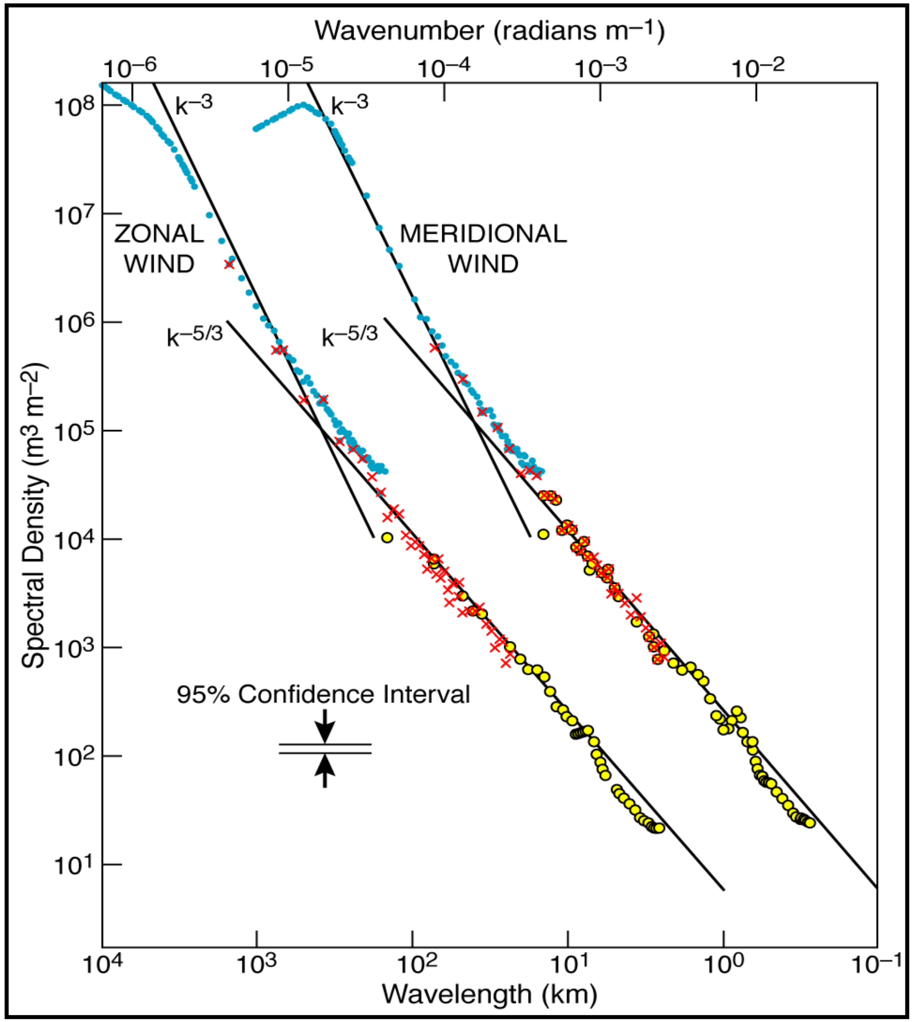
Devil's Bleachers in a 1-D ENSO Model



F.-F. Jin, J.D. Neelin & M. Ghil, *Physica D*, **98**, 442-465, 1996

But deterministic chaos doesn't explain all: there are many other sources of irregularity!

- The energy spectrum of the atmosphere and ocean is "full": all space & time scales are active and they all contribute to forecasting uncertainties.
- Still, one can imagine that the longest & slowest scales contribute most to the longest-term forecasts.
- "One person's signal is another person's noise."



After Nastrom & Gage (*JAS*, 1985)

Climatic uncertainties & moral dilemmas

♥ *… keep today's climate for tomorrow?*

♥ ***Feed the world today***
or…

Davos, Feb. 2008, photos by *TIME Magazine*, 11 Feb. '08;

see also Hillerbrand & Ghil, *Physica D*, 2008, **237**, 2132–2138, doi:10.1016/j.physd.2008.02.015 .